Skillshort

Self-Study Preparation Pack

CA FOUNDATION

Paper 3: Quantitative Aptitude

Chapter-wise Detailed Notes

For January 2027 attempt (useful for May 2027 too) · ICAI BoS Study Material (new scheme) · 18 chapters

100 MCQs · 2 hours | Part A Maths 40 · Part B Reasoning 20 · Part C Stats 40 | Negative marking 0.25 VERIFY | Pass: 40% paper, 50% aggregate

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This subject pack has 6 parts

1 Chapter-wise notes

2 Important topics & weightage

3 Past papers: trends & links

4 Mock tests + solutions

5 Chapter practice questions

6 Last-week plan & strategy

Prepared by Skillshort for self-study. Original explanations and questions; not an official ICAI/ICMAI/ICSI publication. Items marked VERIFY should be checked against the latest official material.

How to use these notes

Quantitative Aptitude is a **speed paper**: 100 MCQs in 120 minutes (72 seconds a question). Each chapter here gives the formulas, the standard question types and **solved examples with shortcuts**. Learn a formula, then immediately do the chapter set in file 05.

File	Use it for
01 Chapter Notes (this file)	Formulas, concepts and solved examples
02 Important Topics & Weightage	Chapter-wise weightage and how many MCQs to expect
03 Past Papers – Trend Analysis & Links	ICAI papers analysis and official links
04 Mock Tests 1–3 with Solutions	Full 100-MCQ, 2-hour tests with answer keys and working
05 Chapter-wise Practice Questions	10 MCQs per chapter with solutions
06 Last-Week Plan & Exam Strategy	Formula sheet, time plan, guessing rules

Paper at a glance

Item	Detail
Marks / time	100 marks, **2 hours**, 100 MCQs of 1 mark each; no reading time
Negative marking	0.25 mark deducted for each wrong answer {v – confirm in your admit card instructions}
Part A – Business Mathematics (40)	I Ratio, indices, logs, equations, inequalities **20–30%**; II Mathematics of finance **30–40%**; III P&C, sequences, sets/functions/limits, calculus **30–50%** (of Part A)
Part B – Logical Reasoning (20)	Number series/coding/odd man out, directions, seating, blood relations – **20–30% each**
Part C – Statistics (40)	V Data presentation, sampling, central tendency & dispersion **45–50%**; VI Probability & theoretical distributions **25–30%**; VII Correlation & regression **10–15%**; VIII Index numbers **10–15%** (of Part C)

STRATEGY Calculator

A simple non-programmable calculator is allowed. Practise the **memory keys** (M+, M−, MR) and the trick for powers: for 1.08^{10} press 1.08 × = repeatedly (9 times) on most calculators; for reciprocals, enter number, ÷, = .

Chapter 1: Ratio and Proportion, Indices and Logarithms

1.1 Ratio and proportion

- **Ratio** a : b = a/b compares two quantities of the same kind.
- **Compound ratio** of a:b and c:d = ac : bd. **Duplicate ratio** of a:b = $a^2 : b^2$. **Sub-duplicate** = √a : √b. **Triplicate** = $a^3 : b^3$.
- **Inverse ratio** of a : b = b : a.
- **Proportion**: a : b = c : d ⇔ ad = bc (product of extremes = product of means).
- **Continued proportion**: a : b = b : c ⇒ b^2 = ac (b is the **mean proportional**). Third proportional to a and b = b^2/a. Fourth proportional to a, b, c = bc/a.

Properties of proportion (if a/b = c/d)

Property	Result
Invertendo	b/a = d/c
Alternendo	a/c = b/d
Componendo	(a + b)/b = (c + d)/d
Dividendo	(a − b)/b = (c − d)/d
Componendo and dividendo	(a + b)/(a − b) = (c + d)/(c − d)

WORKED EXAMPLE **Ratio problems**

(a) Divide ₹72,000 among A, B, C in the ratio 2 : 3 : 4. Total parts 9 → A ₹16,000, B ₹24,000, C ₹32,000.

(b) Mean proportional between 4 and 25 = √100 = **10**.

(c) Two numbers are in ratio 3 : 5; if 9 is added to each, the ratio becomes 3 : 4. Let numbers 3x and 5x: (3x+9)/(5x+9) = 3/4 → 12x + 36 = 15x + 27 → x = 3 → numbers **9 and 15**.

(d) Partners A and B invest ₹40,000 for 12 months and ₹60,000 for 6 months. Profit ratio = 4,80,000 : 3,60,000 = **4 : 3**.

Variation

- **Direct variation**: y ∝ x ⇒ y = kx. **Inverse variation**: y ∝ 1/x ⇒ xy = k. **Joint variation**: z ∝ xy.

WORKED EXAMPLE **Time and work (included in Chapter 1)**

If A can do a job in 10 days and B in 15 days, one day's work = 1/10 + 1/15 = 1/6 → together **6 days**.

If A and B together take 12 days and A alone 20 days, B alone: 1/12 − 1/20 = 1/30 → **30 days**.

Work–wages: wages are shared in the ratio of work done – A (10 days) and B (15 days) working together earn in the ratio 1/10 : 1/15 = 3 : 2.

1.2 Indices (laws of exponents)

Law	Result
Product	$a^m \times a^n = a^{m+n}$
Quotient	$a^m \div a^n = a^{m-n}$
Power	$(a^m)^n = a^{mn}$
Zero index	$a^0 = 1$ ($a \neq 0$)
Negative index	$a^{-n} = 1/a^n$
Fractional index	a^(1/n) = $\sqrt[n]{a}$; a^(m/n) = $(\sqrt[n]{a})^m$
Product of bases	$(ab)^n = a^n b^n$; $(a/b)^n = a^n/b^n$
Equal powers	If $a^x = a^y$ ($a \neq 1$) then x = y

WORKED EXAMPLE **Indices**

(a) Simplify (27)^(2/3) × (16)^(−1/4) = 9 × 1/2 = **4.5**.

(b) If $2^x = 3^y = 6^{(-z)}$, then 1/x + 1/y + 1/z = **0** (let each = k: 2 = k^(1/x), 3 = k^(1/y), 6 = k^(−1/z); 2 × 3 = 6 → 1/x + 1/y = −1/z).

(c) If x = 3^(1/3) + 3^(−1/3), then $3x^3 - 9x$ = **10** (cube: $x^3 = 3 + 1/3 + 3x \rightarrow 3x^3 = 10 + 9x$).

1.3 Logarithms

If $a^x = N$ ($a > 0$, $a \neq 1$, $N > 0$), then **$x = \log_a N$**. Common logs have base 10; natural logs base e.

Rule	Result
Product	log (mn) = log m + log n
Quotient	log (m/n) = log m − log n
Power	$\log m^n = n \log m$
Base change	$\log_a N = \log_b N \div \log_b a$; $\log_a b \times \log_b a = 1$
Special values	$\log_a a = 1$; $\log_a 1 = 0$; a^($\log_a N$) = N
Power of base	$\log_{(a^k)} N = (1/k) \log_a N$

WORKED EXAMPLE **Logarithms**

(a) $\log_2 64 = 6$ ($2^6 = 64$). log √2 of 8 = $\log_2 8 \div (1/2)$ = **6**.

(b) Solve $\log_x 4 + \log_x 16 + \log_x 64 = 12 \rightarrow \log_x (4 \times 16 \times 64) = \log_x 4096 = 12 \rightarrow x^{12} = 4096 = 2^{12} \rightarrow$ **x = 2**.

(c) log 2 = 0.3010, log 3 = 0.4771 → log 72 = 3 log 2 + 2 log 3 = 0.9030 + 0.9542 = **1.8572**.

(d) Number of digits in 2^{50}: 50 × 0.3010 = 15.05 → characteristic 15 → **16 digits**.

(e) Antilog: if log x = 2.4771, x = antilog 2.4771 = 10^2 × 10^0.4771 = **300**.

MEMORY TRICK **Characteristic and mantissa**

For a number > 1, characteristic = (number of digits before the decimal point) − 1. For 0.00345, characteristic = −3 (written $\bar{3}$). Mantissa is always positive and taken from tables.

Chapter 2: Equations

2.1 Linear and simultaneous equations

- Linear equation in one variable: $ax + b = 0 \rightarrow x = -b/a$.
- **Simultaneous equations** in two/three variables: solve by **elimination**, substitution or **Cramer's rule** (determinants). For MCQs, **substitute the options** – fastest.

WORKED EXAMPLE Simultaneous equations

(a) $2x + 3y = 13$, $3x - y = 3 \rightarrow$ from second, $y = 3x - 3 \rightarrow 2x + 9x - 9 = 13 \rightarrow x = 2$, $y = 3$.

(b) $x + y + z = 6$, $x - y + z = 2$, $2x + y - z = 1 \rightarrow$ subtract first two: $2y = 4 \rightarrow y = 2$; then $x + z = 4$ and $2x - z = -1 \rightarrow 3x = 3 \rightarrow x = 1$, $z = 3$.

(c) Business: cost of 3 pens and 2 notebooks is ₹170; 2 pens and 3 notebooks ₹180 → adding: $5(p + n) = 350 \rightarrow p + n = 70$; subtracting: $n - p = 10 \rightarrow n = 40$, $p = 30$.

2.2 Quadratic equations

$ax^2 + bx + c = 0$ ($a \neq 0$). Roots: **$x = [-b \pm \sqrt{(b^2 - 4ac)}] \div 2a$**.

Item	Result
Sum of roots $\alpha + \beta$	$-b/a$
Product $\alpha\beta$	c/a
Discriminant $D = b^2 - 4ac$	$D > 0$ real & unequal; $D = 0$ real & equal; $D < 0$ imaginary; D a perfect square (a, b, c rational) → rational roots
Equation from roots	$x^2 - (\alpha + \beta)x + \alpha\beta = 0$
Useful identities	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$; $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$; $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
Irrational roots	Occur in conjugate pairs $(p + \sqrt{q}, p - \sqrt{q})$

WORKED EXAMPLE Quadratics

(a) $x^2 - x - 6 = 0 \rightarrow \alpha + \beta = 1$, $\alpha\beta = -6 \rightarrow \alpha^2 + \beta^2 = 1 + 12 = 13$; $\alpha^3 + \beta^3 = 1 + 18 = 19$.

(b) Roots of $2x^2 - 6x + 3 = 0$ are α, β. Equation with roots α/β and β/α: sum $= (\alpha^2 + \beta^2)/\alpha\beta = (9 - 3)/1.5 = 4$; product $= 1 \rightarrow$ **$x^2 - 4x + 1 = 0$**.

(c) If one root of $x^2 - 5x + k = 0$ is 2, then $4 - 10 + k = 0 \rightarrow k = 6$; other root 3.

2.3 Cubic equations

For $ax^3 + bx^2 + cx + d = 0$ with roots α, β, γ: $\alpha + \beta + \gamma = -b/a$; $\alpha\beta + \beta\gamma + \gamma\alpha = c/a$; $\alpha\beta\gamma = -d/a$. Find one root by trial (±1, ±2, factors of d), then divide to get a quadratic.

WORKED EXAMPLE $x^3 - 6x^2 + 11x - 6 = 0 \rightarrow x = 1$ is a root $\rightarrow (x - 1)(x^2 - 5x + 6) = 0 \rightarrow$ roots 1, 2, 3.

2.4 Business applications

Break-even: Revenue = Cost → $p \cdot x = F + v \cdot x \rightarrow x = F \div (p - v)$. Profit equation $P(x) = R(x) - C(x)$. Age problems, number problems and mixture problems reduce to linear/quadratic equations.

Chapter 3: Linear Inequalities

- A linear inequality in one variable: ax + b < 0 (or ≤, >, ≥). Rules: add/subtract any number; multiply/divide by a **positive** number – sign unchanged; by a **negative** number – **sign reverses**.
- In two variables, ax + by ≤ c represents a **half-plane**. Draw the line ax + by = c (solid for ≤/≥, dashed for </>); test the origin (0, 0) to pick the side.
- The **feasible region** of a system is the common region; its **corner points** give the maximum/minimum of a linear objective function (linear programming idea).

WORKED EXAMPLE **Linear inequalities**

(a) $5x/3 \le x/6 - 5$ → multiply by 6: $10x \le x - 30$ → $9x \le -30$ → **$x \le -10/3$**, i.e., $(-\infty, -10/3]$.

(b) $-3 < 2x + 1 \le 7$ → $-4 < 2x \le 6$ → **$-2 < x \le 3$**.

(c) Region $x \ge 0$, $y \ge 0$, $x + y \le 4$, $x + 2y \le 6$: corner points (0,0), (4,0), (2,2), (0,3). Max of $Z = 3x + 4y$: values 0, 12, 14, 12 → **max 14 at (2, 2)**.

MEMORY TRICK **For "which point satisfies the system" MCQs, substitute each option point into all inequalities – it takes 20 seconds.**

Chapter 4: Mathematics of Finance

4.1 Simple interest

Interest is the payment for the use of borrowed money. The original sum is the **principal (P)**; principal + interest is the **amount (A)**.

FORMULA Simple interest

SI = P × R × T ÷ 100

A = P + SI = P (1 + RT/100)

P = 100 × SI ÷ (R × T); R = 100 × SI ÷ (P × T); T = 100 × SI ÷ (P × R)

WORKED EXAMPLE Simple interest

(a) Find SI on ₹50,000 at 8% p.a. for 3 years. SI = 50,000 × 8 × 3 ÷ 100 = **₹12,000**; Amount = ₹62,000.

(b) In how many years will a sum double at 10% simple interest? SI = P → P = P × 10 × T/100 → **T = 10 years**.

(c) A sum amounts to ₹13,200 in 2 years and ₹14,400 in 4 years at SI. Interest for 2 years = ₹1,200 → per year ₹600. Principal = 13,200 − 1,200 = ₹12,000; R = 600 × 100 ÷ 12,000 = **5%**.

4.2 Compound interest, effective rate and present value

Under **compound interest (CI)**, interest is added to the principal at the end of each period, and interest for the next period is calculated on the increased amount.

FORMULA Compound interest

A = P (1 + r/100)^n ; CI = A − P

Compounded k times a year: A = P (1 + r/(100k))^(nk)

Effective rate of interest: E = (1 + r/(100k))^k − 1

Depreciation (reducing balance): Value after n years = P (1 − r/100)^n

Population growth: P_n = P_0 (1 + r/100)^n

WORKED EXAMPLE Compound interest

(a) ₹1,00,000 at 10% p.a. compounded annually for 2 years: A = 1,00,000 × $(1.1)^2$ = ₹1,21,000; **CI = ₹21,000** (SI would be ₹20,000; difference ₹1,000 = $P(r/100)^2$).

(b) Compounded half-yearly: A = 1,00,000 × $(1.05)^4$ = ₹1,21,550.63; CI = ₹21,550.63.

(c) Effective rate of 12% p.a. compounded monthly: $(1.01)^{12}$ − 1 = 0.1268 → **12.68%**.

(d) A machine costing ₹2,00,000 depreciates at 10% p.a. (reducing balance). Value after 3 years = 2,00,000 × $(0.9)^3$ = **₹1,45,800**.

MEMORY TRICK Difference between CI and SI for 2 years = P (r/100)². For 3 years = P (r/100)² (3 + r/100).

Present value and discounting

FORMULA Present value

PV = A ÷ (1 + r/100)^n

WORKED EXAMPLE Present value

What sum invested now at 8% p.a. compound will become ₹1,00,000 in 3 years? PV = 1,00,000 ÷ $(1.08)^3$ = 1,00,000 ÷ 1.259712 = **₹79,383** (approx).

4.3 Annuities, perpetuity, sinking fund and EMI

An **annuity** is a series of **equal payments** made at **equal intervals** of time (EMIs, insurance premiums, pensions, SIP instalments, rent).

Type	Meaning
Annuity regular (ordinary/immediate)	Payment at the **end** of each period
Annuity due	Payment at the **beginning** of each period
Perpetuity	Payments continue forever
Deferred annuity	Payments start after a certain number of periods
Contingent annuity	Depends on an uncertain event (life annuity)

FORMULA Annuity formulas (i = rate per period as a decimal, n = number of periods)

Future value of annuity regular: FV = A × [(1 + i)^n − 1] ÷ i

Present value of annuity regular: PV = A × [1 − (1 + i)^−n] ÷ i

Annuity due: FV(due) = FV(regular) × (1 + i); PV(due) = PV(regular) × (1 + i)

Perpetuity: PV = A ÷ i

EMI on a loan: EMI = P × i × (1 + i)^n ÷ [(1 + i)^n − 1]

Sinking fund instalment: A = FV × i ÷ [(1 + i)^n − 1]

WORKED EXAMPLE Annuities

(a) ₹10,000 deposited at the end of each year for 5 years at 10%: FV = 10,000 × $(1.1^5 - 1)/0.1$ = 10,000 × 6.1051 = **₹61,051**.

(b) PV of ₹10,000 received at the end of each year for 5 years at 10%: 10,000 × $(1 - 1.1^{-5})/0.1$ = 10,000 × 3.7908 = **₹37,908**.

(c) If the payments are at the beginning of each year (annuity due): FV = 61,051 × 1.1 = ₹67,156.

(d) EMI on a loan of ₹5,00,000 at 12% p.a. for 2 years (monthly): i = 0.01, n = 24; $(1.01)^{24}$ = 1.26973. EMI = 5,00,000 × 0.01 × 1.26973 ÷ 0.26973 = **₹23,537** (approx).

(e) Perpetuity of ₹6,000 a year at 8%: PV = 6,000 ÷ 0.08 = **₹75,000**.

FORMULA Growing perpetuity and deferred annuity

Growing perpetuity (first payment A at end of year 1, growth g): PV = A ÷ (i − g), i > g

Deferred annuity (first payment after m periods): PV = A × [1 − (1 + i)^−n] ÷ i × (1 + i)^−m

4.4 Net present value, returns and CAGR

- **NPV** = PV of cash inflows − PV of cash outflows (initial investment). Accept a project if NPV > 0.
- **Nominal rate of return** = (Ending value − Beginning value + income) ÷ Beginning value. **Real rate** ≈ nominal − inflation; exactly (1 + nominal)/(1 + inflation) − 1.
- **CAGR** = (Ending value ÷ Beginning value)^(1/n) − 1.
- **Effective rate** E = (1 + r/k)^k − 1 (r nominal annual rate as decimal, k compounding periods per year).

WORKED EXAMPLE **NPV, CAGR, real return**

(a) Investment ₹1,00,000; inflows ₹40,000 a year for 3 years; rate 10%. PV factor of annuity (3 yrs, 10%) = 2.4869 → PV inflows = 99,474 → **NPV = −526** → reject.

(b) Investment grows from ₹50,000 to ₹80,526 in 5 years → CAGR = (1.61052)^(1/5) − 1 = **10%**.

(c) Nominal return 12%, inflation 5% → real return = 1.12/1.05 − 1 = **6.67%** (approximately 7%).

(d) Sinking fund: to accumulate ₹5,00,000 in 5 years at 8%, annual instalment = 5,00,000 × 0.08 ÷ (1.08^5 − 1) = 5,00,000 × 0.08 ÷ 0.469328 = **₹85,228**.

MEMORY TRICK **Rule of 72 – money doubles in about 72 ÷ rate years (at 8%, ≈ 9 years). Exact doubling time at CI: n = log 2 ÷ log(1 + r).**

Chapter 5: Permutations and Combinations

5.1 Fundamental principles and factorial

- **Multiplication rule:** if one job can be done in m ways and another in n ways, both together in **m × n** ways. **Addition rule:** either of two exclusive jobs in **m + n** ways.
- n! = n × (n − 1) × ... × 1; 0! = 1.

5.2 Permutations (arrangements)

Situation	Formula
r objects out of n distinct	${}^nP_r = n! \div (n - r)!$
All n distinct	n!
With repetition allowed (r places)	n^r
n objects with p alike, q alike...	n! ÷ (p! q! ...)
Circular (n distinct)	(n − 1)!
Circular where clockwise = anticlockwise (necklace, garland)	(n − 1)! ÷ 2
Two particular objects always together	(n − 1)! × 2!
Two particular objects never together	n! − (n − 1)! × 2!

5.3 Combinations (selections)

${}^nC_r = n! \div [r!\,(n - r)!]$; ${}^nC_r = {}^nC_{n-r}$; ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$; ${}^nP_r = {}^nC_r \times r!$; total selections of at least one from n = $2^n - 1$.

Situation	Formula
Diagonals of an n-sided polygon	${}^nC_2 - n$
Lines through n points (no 3 collinear)	${}^nC_2$
Triangles from n points (m collinear)	${}^nC_3 - {}^mC_3$
Handshakes among n persons	${}^nC_2$
A particular object always included	${}^{n-1}C_{r-1}$; always excluded ${}^{n-1}C_r$

WORKED EXAMPLE P & C

(a) Arrangements of the letters of "ACCOUNTANT": 10 letters – A 2, C 2, N 2, T 2, O 1, U 1 → 10! ÷ $(2!)^4$ = 3,628,800 ÷ 16 = **2,26,800**.

(b) 7 people around a round table: 6! = **720**; two particular people together: 5! × 2 = **240**.

(c) Committee of 5 from 6 men and 4 women with at least 2 women: ${}^4C_2 \cdot {}^6C_3 + {}^4C_3 \cdot {}^6C_2 + {}^4C_4 \cdot {}^6C_1$ = 120 + 60 + 6 = **186**.

(d) ${}^nP_r = r! \times {}^nC_r$, so ${}^nP_3 = 6 \times {}^nC_3$ for every n. If ${}^nC_2 = 45$, n(n − 1) = 90 → **n = 10**.

(e) 4-digit numbers from 1–9 without repetition: ${}^9P_4$ = **3,024**; with repetition: 9^4 = 6,561.

Chapter 6: Sequence and Series

6.1 Arithmetic progression (AP)

a, a + d, a + 2d ... ; **$t_n = a + (n - 1)d$**; **$S_n = n/2\ [2a + (n - 1)d] = n/2\ (a + l)$**. Arithmetic mean of a and b = (a + b)/2. Three numbers in AP: a − d, a, a + d.

6.2 Geometric progression (GP)

a, ar, ar^2 ... ; **$t_n = ar^{n-1}$**; **$S_n = a(r^n - 1)/(r - 1)$** ($r \neq 1$); infinite GP ($|r| < 1$): **$S\infty = a/(1 - r)$**. Geometric mean of a, b = √(ab). Three numbers in GP: a/r, a, ar.

6.3 Relation between AM, GM (and HM)

For positive a, b: **AM ≥ GM ≥ HM**; GM^2 = AM × HM; HM = 2ab/(a + b).

6.4 Special series

Series	Sum
1 + 2 + ... + n	n(n + 1)/2
$1^2 + 2^2 + ... + n^2$	n(n + 1)(2n + 1)/6
$1^3 + 2^3 + ... + n^3$	$[n(n + 1)/2]^2$
Sum of first n odd numbers	n^2
0.3 + 0.33 + 0.333 + ... n terms	$(1/3)[n - (1/9)(1 - 10^{-n})]$

WORKED EXAMPLE Sequences

(a) 15th term of 3, 7, 11 ... = 3 + 14 × 4 = **59**; sum of 15 terms = 15/2 (3 + 59) = **465**.

(b) Sum of all two-digit numbers divisible by 3: 12, 15 ... 99 → n = 30 → S = 30/2 (12 + 99) = **1,665**.

(c) 8th term of 2, 6, 18 ... = 2×3^7 = **4,374**. S∞ of 1 + 1/2 + 1/4 + ... = **2**.

(d) Business: salary ₹30,000 rising ₹2,000 a year – total earnings in 10 years = 10/2 [60,000 + 9 × 2,000] = **₹3,90,000**.

Chapter 7: Sets, Relations, Functions; Limits and Continuity

7.1 Sets

A **set** is a well-defined collection of distinct objects. Notation: A = {1, 2, 3} (roster form) or A = {x : x is a natural number less than 4} (set-builder form).

Term	Meaning
Empty (null) set ⌀	No element
Singleton	One element
Finite / infinite	Countable number of elements / not
Subset A ⊆ B	Every element of A is in B; number of subsets of a set with n elements = 2^n
Proper subset	A ⊂ B and A ≠ B
Power set P(A)	Set of all subsets of A
Universal set U	Set containing all elements under consideration
Union A ∪ B	Elements in A or B or both
Intersection A ∩ B	Elements in both
Difference A − B	Elements in A but not in B
Complement A'	Elements in U but not in A
Disjoint sets	A ∩ B = ⌀

FORMULA Set laws and counting

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

De Morgan's laws: $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$

WORKED EXAMPLE Set application

In a class of 100 students, 60 study Accounts, 50 study Economics and 20 study both.

At least one = 60 + 50 − 20 = **90**. Neither = 100 − 90 = **10**. Only Accounts = 60 − 20 = **40**; only Economics = **30**.

EXAM TIP Draw a Venn diagram for every set problem – fill the intersection first, then the "only" regions.

7.2 Relations

- **Cartesian product** A × B = {(a, b): a ∈ A, b ∈ B}; n(A × B) = n(A) × n(B).
- A **relation** from A to B is any subset of A × B; number of relations = 2^(mn).
- Types on a set A: **reflexive** (a, a) ∈ R ∀a; **symmetric** (a, b) ∈ R ⇒ (b, a) ∈ R; **transitive** (a, b), (b, c) ∈ R ⇒ (a, c) ∈ R; **equivalence** = all three. **Identity relation** {(a, a)}; **inverse relation** R^{-1} = {(b, a)}.
- **Domain** = set of first elements; **range** = set of second elements.

7.3 Functions

- A **function** f: A → B assigns to each element of A **exactly one** element of B. Types: **one-one (injective)**, **onto (surjective)** – range = co-domain, **bijective** (both), constant, identity.
- Composite: (f ∘ g)(x) = f(g(x)). Inverse exists only for bijections. Number of functions from A (m) to B (n) = n^m.

WORKED EXAMPLE **$f(x) = 2x + 3, g(x) = x^2 \rightarrow f(g(2)) = f(4) = 11$; $g(f(2)) = g(7) = 49$. Inverse of f: $y = 2x + 3 \rightarrow f^{-1}(x) = (x - 3)/2$.**

7.4 Limits and continuity

Standard limit	Value
lim $(x^n - a^n)/(x - a)$, $x \to a$	$n \cdot a^{n-1}$
lim $(1 + 1/n)^n$, $n \to \infty$	e
lim $(1 + x)^{(1/x)}$, $x \to 0$	e
lim $(e^x - 1)/x$, $x \to 0$	1
lim $(a^x - 1)/x$, $x \to 0$	$\log_e a$
lim $\log(1 + x)/x$, $x \to 0$	1

- For 0/0 forms, **factorise** and cancel, or rationalise. For $x \to \infty$, divide by the highest power.
- f is **continuous at x = a** if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.

WORKED EXAMPLE **lim $(x^2 - 9)/(x - 3)$ as $x \to 3$ = lim $(x + 3)$ = 6. lim $(3x^2 + 2x)/(5x^2 - 1)$ as $x \to \infty$ = 3/5.**

Chapter 8: Basic Applications of Differential and Integral Calculus

8.1 Differentiation – rules

Function	Derivative
x^n	$n\,x^{n-1}$
constant	0
e^x ; a^x	e^x ; $a^x \log_e a$
log x	1/x
u ± v	u′ ± v′
u·v (product)	u v′ + v u′
u/v (quotient)	(v u′ − u v′)/v^2
f(g(x)) (chain)	f′(g(x)) · g′(x)

- **Second derivative** d^2y/dx^2; **implicit** and **parametric** differentiation: dy/dx = (dy/dt)/(dx/dt).

8.2 Applications in business

- **Marginal cost** MC = dC/dx; **marginal revenue** MR = dR/dx; average cost AC = C/x.
- **Maxima/minima:** set f′(x) = 0; if f″(x) < 0 → maximum; f″(x) > 0 → minimum. Profit is maximum where MR = MC and slope condition holds.
- **Elasticity of demand** = −(p/x)(dx/dp).

WORKED EXAMPLE Differential calculus

(a) $C(x) = 0.5x^2 + 20x + 500$ → MC = x + 20; at x = 30, **MC = ₹50**.

(b) $R(x) = 100x - 2x^2$ → MR = 100 − 4x; R maximum when 100 − 4x = 0 → x = 25; R″ = −4 < 0 ✔ → max revenue **₹1,250**.

(c) $y = x^3 - 6x^2 + 9x$ → $y' = 3x^2 - 12x + 9 = 0$ → x = 1 or 3; y″ = 6x − 12: at x = 1, −6 (max, y = 4); at x = 3, +6 (min, y = 0).

8.3 Integration

Integral	Result
$\int x^n\,dx$	$x^{n+1}/(n+1) + c$ $(n \neq -1)$
$\int 1/x\,dx$	log x + c
$\int e^x\,dx$	$e^x + c$
$\int a^x\,dx$	$a^x/\log a + c$
$\int e^x [f(x) + f'(x)]\,dx$	$e^x f(x) + c$
Definite $\int_a^b f(x)\,dx$	F(b) − F(a)

- Methods: substitution, **by parts** $\int u\,v\,dx = u\int v\,dx - \int[u' \int v\,dx]\,dx$, partial fractions.
- **Applications:** total cost C = ∫ MC dx + fixed cost; total revenue R = ∫ MR dx (R(0) = 0); area under a curve.

WORKED EXAMPLE Integral calculus

(a) $\int_0^2 (3x^2 + 2x)\,dx = [x^3 + x^2]_0^2 = 8 + 4 = \mathbf{12}$.

(b) MC = 4x + 5, fixed cost ₹100 → $C(x) = 2x^2 + 5x + 100$; cost of 10 units = 200 + 50 + 100 = **₹350**.

(c) $\int x\,e^x\,dx = x\,e^x - e^x + c$ (by parts).

Chapter 9: Number Series, Coding–Decoding and Odd Man Out

9.1 Number series – patterns to test (in this order)

1. **Differences** (constant, increasing by a constant, alternating).
2. **Ratios** (×2, ×3, ×2 + 1, ×1.5).
3. **Squares/cubes** and ±1 (1, 4, 9, 16 / 2, 9, 28, 65 = $n^3 + 1$).
4. **Primes** (2, 3, 5, 7, 11, 13 ...).
5. **Alternate series** – two series interleaved.
6. **Mixed operations** (×2 + 1, ×3 − 1 ...).

WORKED EXAMPLE **Number series**

(a) 3, 8, 15, 24, 35, ? → differences 5, 7, 9, 11, 13 → **48** ($n^2 - 1$ pattern: 2^2-1 ... 7^2-1).

(b) 2, 6, 12, 20, 30, ? → n(n + 1) → **42**.

(c) 5, 11, 23, 47, 95, ? → ×2 + 1 → **191**.

(d) 1, 3, 4, 8, 15, 27, ? → each term = sum of previous three → **50**.

9.2 Letter series and coding–decoding

- Learn **positions** (A = 1 ... Z = 26) and reverse positions (A = 26 ... Z = 1): **EJOTY** = 5, 10, 15, 20, 25.
- Coding types: **letter shift** (+1, −2), **reverse order**, **position-value coding**, **substitution**, **number coding**.

WORKED EXAMPLE **Coding**

(a) If MANGO is coded NBOHP (+1 each), APPLE → **BQQMF**.

(b) If CAT = 24 (3 + 1 + 20), DOG = 4 + 15 + 7 = **26**.

(c) If in a code "sky is blue" = "ta na pa" and "blue is deep" = "na ka pa", then "deep" = **ka** (common words "is blue" = "na pa").

9.3 Odd man out

Look for the property shared by all but one: prime/composite, perfect square/cube, even/odd, divisibility, vowels, category (fruits vs vegetables). Example: 27, 64, 125, 144, 216 → **144** (not a cube).

Chapter 10: Direction Sense Tests

- Draw a **compass** (N up, E right). Right turn = clockwise 90°, left turn = anticlockwise 90°.
- **Shortest distance** = $\sqrt{(x^2 + y^2)}$ (Pythagoras) using net east–west and net north–south displacement.
- Shadows: in the **morning** the sun is in the east, so shadows fall to the **west**; in the **evening**, shadows fall to the **east**.

WORKED EXAMPLE **Directions**

(a) Ravi walks 5 km north, turns right and walks 3 km, turns right and walks 9 km. Net: east 3, south 4 → distance **5 km** towards **south-east**.

(b) Facing north, turn 90° clockwise, then 135° anticlockwise → net 45° anticlockwise from north → facing **north-west**.

(c) In the evening, Meena's shadow falls to her right → she is facing **south** (shadow east = right ⇒ facing south).

Chapter 11: Seating Arrangements

- **Linear (row):** fix the person with the most definite clue first; "immediate left/right" means adjacent; facing north – left of a person is west; facing south – left is east.
- **Circular facing centre:** clockwise is the person's **left**; with n persons, the person opposite is n/2 places away.
- Make a **table** of possibilities; eliminate with negative clues; check every option.

WORKED EXAMPLE **Six friends A–F sit in a circle facing the centre. B is to the immediate left of A; C is opposite A; D is between C and E… Draw the circle with A at the bottom, B next clockwise (A's left), C at top, then place D and E around C. Typical questions: "Who sits opposite B?", "Who is second to the right of D?".**

EXAM TIP **Write the final arrangement in the margin and answer all questions on that set from it – sets of 3–5 questions are common.**

Chapter 12: Blood Relations

- Use symbols: **+** male, **–** female, **=** married couple, vertical line for generations, horizontal for siblings.
- Common relations: father's or mother's brother – uncle (paternal/maternal); brother's son – nephew; son's wife – daughter-in-law; wife's brother – brother-in-law; father's father – grandfather.
- **Coded relations:** "A + B means A is the father of B", "A × B means A is the sister of B" – decode step by step from right to left.

WORKED EXAMPLE **Blood relations**

(a) Pointing to a man, Rita said, "His mother is the only daughter of my mother." The man's mother is Rita herself → Rita is his **mother**.

(b) A is B's sister; C is B's mother; D is C's father; E is D's mother. A is related to D as **granddaughter**.

(c) If P + Q means P is Q's husband, P − Q means P is Q's sister and P × Q means P is Q's son, then in M × N + O, M is the son of N and N is the husband of O → M is the **son** of O.

Chapter 13: Statistical Description of Data and Sampling

13.1 Statistics, collection and presentation

Statistics in the plural sense means numerical facts (data); in the singular sense it is the **science of collecting, organising, presenting, analysing and interpreting** numerical data. Croxton and Cowden: "Statistics may be defined as the collection, presentation, analysis and interpretation of numerical data."

Functions

Simplifies complex data, facilitates comparison, helps in forecasting and policy-making, tests hypotheses, and enlarges individual knowledge.

Limitations

Studies only aggregates (not individuals), deals only with quantitative data, results are true only on average, can be misused, and requires expertise.

Uses in business

Market research, quality control, sales forecasting, production planning, financial analysis, inventory control, HR (wage analysis), and insurance (actuarial).

Basis	Primary data	Secondary data
Meaning	Collected for the first time by the investigator for a specific purpose	Already collected by someone else and used by the investigator
Originality	Original	Not original
Cost and time	Expensive, time-consuming	Cheaper, quicker
Suitability	Exactly suited to the purpose	May need adjustment
Examples	Survey of consumers by a company	Census reports, RBI bulletins, NSO publications

Methods of collecting primary data

1. **Direct personal investigation** – investigator collects data personally (small area, accurate).
2. **Indirect oral investigation** – from witnesses or third parties (commissions of inquiry).
3. **Information through local correspondents** – newspapers, agencies.
4. **Questionnaires mailed** to informants – wide area, low cost, but low response.
5. **Schedules filled by enumerators** – e.g. Census of India.
6. Online surveys (Google Forms etc.) – modern method.

Qualities of a good questionnaire

Short, simple questions; logical order; avoid personal or confusing questions; avoid leading questions; include instructions; pre-test (pilot survey); mostly closed-ended (yes/no, multiple choice).

Sources of secondary data

Published: government publications (Census, NSO, Economic Survey, RBI Bulletin), international bodies (UN, IMF, World Bank), journals, reports of committees, trade associations. **Unpublished**: records of firms, research scholars.

Census vs sample method

Basis	Census (complete enumeration)	Sample method
Coverage	Every unit of the population	Representative part
Cost and time	High	Low
Accuracy	High (if done well)	Depends on sample; sampling error
Suitable	Small, heterogeneous population; detailed information	Large, homogeneous population; destructive testing

Sampling methods

- **Random (probability) sampling** – simple random (lottery, random number tables), stratified, systematic, cluster, multi-stage.
- **Non-random sampling** – judgement (purposive), convenience, quota.

13.2 Classification, tabulation and frequency distribution

Classification is the process of arranging data into groups or classes according to common characteristics – **geographical** (by place), **chronological** (by time), **qualitative** (by attributes – sex, literacy), **quantitative** (by measurable characteristics – income, marks).

Frequency distribution – terms

Term	Meaning
Class interval	Group such as 10–20
Class limits	Lower (10) and upper (20) limits
Class width (magnitude)	Upper limit – lower limit
Mid-value (class mark)	(Lower + upper limit) ÷ 2
Frequency	Number of observations in a class
Exclusive method	10–20, 20–30 (upper limit excluded)
Inclusive method	10–19, 20–29 (both limits included)
Open-end class	"Below 10" or "60 and above"
Cumulative frequency	Running total of frequencies (less than / more than type)
Relative frequency	Class frequency ÷ total frequency

REMEMBER **To convert inclusive classes into exclusive classes, subtract half the gap (0.5) from lower limits and add 0.5 to upper limits: 10–19 becomes 9.5–19.5. Do this before calculating median and mode.**

Sturges' rule

FORMULA **Number of classes (Sturges)**

$k = 1 + 3.322 \log_{10} N$; class width = Range ÷ k

Tabulation

Tabulation is the systematic arrangement of data in rows and columns. Parts of a table: **table number, title, head note, stub (row headings), caption (column headings), body, foot note, source note**.

13.3 Diagrams and graphs

Diagram / graph	Use
Simple bar diagram	Compare one variable across categories/years
Multiple bar diagram	Compare two or more variables side by side
Sub-divided (component) bar	Show parts of a total
Percentage bar diagram	Components as % of total
Pie diagram	Components of a total as sectors of a circle (angle = component ÷ total × 360°)
Pictogram, cartogram	Pictures / maps
Histogram	Continuous frequency distribution – adjacent rectangles; used to find **mode** graphically
Frequency polygon	Joining mid-points of histogram tops
Frequency curve	Smoothed polygon

Diagram / graph	Use
Ogive (cumulative frequency curve)	Less-than and more-than ogives; their intersection gives the **median**; also used for quartiles
Line graph / time-series graph (historigram)	Changes over time
Lorenz curve	Inequality of income distribution

WORKED EXAMPLE **Pie diagram angles**

Monthly family budget: Food ₹12,000, Rent ₹6,000, Education ₹4,000, Others ₹8,000 (total ₹30,000).

Angles: Food 12/30 × 360 = 144°; Rent 72°; Education 48°; Others 96°. Total 360° ✓.

EXAM TIP **Always give a title, scale, axis labels and source in diagrams – they carry marks.**

NOTE **Facts ICAI asks as one-liners**

Ogives (cumulative frequency curves) locate **median and quartiles** graphically; histogram locates **mode**; the intersection of less-than and more-than ogives gives the **median**. A **pie chart** shows components of a total (angle = value/total × 360°). Frequency density = frequency ÷ class width (used for unequal classes). **Class mark** = (lower + upper limit)/2.

13.4 Sampling

Term	Meaning
Population / universe	Aggregate of all units under study
Sample	Part of the population selected for study
Parameter vs statistic	Measure of the population (μ, σ) vs measure of a sample ($\bar{x}$, s)
Sampling frame	List of all units from which the sample is drawn
Sampling distribution & standard error	Distribution of a statistic over all samples; SE = its standard deviation (SE of mean = σ/√n)
Census (complete enumeration) vs sample survey	Census – every unit, accurate but costly and slow; sample – quicker, cheaper, allows detailed enquiry, the only option for destructive testing
Principles	**Statistical regularity** (a large random sample shows the characteristics of the population) and **inertia of large numbers** (large groups are stable)

Types of sampling: (1) **probability sampling** – simple random (lottery, random numbers; with or without replacement), stratified (homogeneous strata, sample from each), systematic (every kth unit), multi-stage, cluster; (2) **non-probability** – purposive/judgment, quota, convenience; (3) **mixed** – systematic is often called mixed sampling.

Errors: sampling errors – arise because only a part is studied (reduced by larger samples); **non-sampling errors** – faulty questionnaire, response bias, processing/recording errors – present in census too.

WORKED EXAMPLE **Number of samples of size n from N without replacement = ${}^{N}C_{n}$; with replacement = N^{n}. For N = 5, n = 2: 10 and 25.**

Chapter 14: Measures of Central Tendency and Dispersion

14.1 Averages

An **average** is a single value that represents the whole data set; it lies somewhere in the centre, hence called a **measure of central tendency**.

Qualities (Yule): rigidly defined, easy to understand and calculate, based on all observations, capable of further algebraic treatment, least affected by sampling fluctuations, not unduly affected by extreme values.

FORMULA Arithmetic mean

Individual series: $\bar{X} = \Sigma X / N$

Discrete series: $\bar{X} = \Sigma fx / N$

Continuous series (direct): $\bar{X} = \Sigma fm / N$ (m = mid-value)

Shortcut: $\bar{X} = A + \Sigma fd / N$ (d = m − A)

Step deviation: $\bar{X} = A + (\Sigma fd' / N) \times h$ (d' = (m − A)/h)

Combined mean: $\bar{X}_{12} = (N_1\bar{X}_1 + N_2\bar{X}_2) / (N_1 + N_2)$

Weighted mean: $\bar{X}w = \Sigma wx / \Sigma w$

Properties of AM

- Sum of deviations from the mean is **zero**: $\Sigma(X - \bar{X}) = 0$.
- Sum of squared deviations from the mean is **minimum**.
- Combined mean can be calculated.

Merits and demerits

Merits: rigidly defined, based on all values, algebraic treatment, stable. Demerits: affected by **extreme values**, cannot be found for open-end classes without assumption, may not be an actual value (average 2.5 children).

Median is the **middle value** of the data arranged in ascending or descending order; it divides the data into two equal parts.

FORMULA Median

Individual/discrete: Median = size of (N + 1)/2 th item

Continuous: Median = $L + [(N/2 - cf) / f] \times h$

L = lower limit of median class; cf = cumulative frequency of the class before; f = frequency of median class; h = class width

Partition values: **quartiles** (Q1, Q2 = median, Q3), **deciles** (D1–D9), **percentiles** (P1–P99).

FORMULA Quartiles (continuous series)

$Q1 = L + [(N/4 - cf)/f] \times h$; $Q3 = L + [(3N/4 - cf)/f] \times h$

Merits: not affected by extreme values; can be found for open-end classes; can be found graphically (ogives). Demerits: not based on all values; not capable of algebraic treatment.

Mode is the value that occurs **most frequently** – the most typical value (e.g. most common shoe size).

FORMULA Mode (continuous series)

Mode = $L + [(f_1 - f_o) / (2f_1 - f_o - f_2)] \times h$

f_1 = frequency of modal class; f_o = frequency of preceding class; f_2 = frequency of succeeding class

Empirical relation (moderately skewed): Mode = 3 Median − 2 Mean

When frequencies are irregular, **grouping method** is used to locate the modal class. A distribution may be unimodal, bimodal or multimodal.

Marks	Students (f)	Mid-value (m)	fm	Cumulative f
0–10	5	5	25	5
10–20	8	15	120	13
20–30	15	25	375	28
30–40	7	35	245	35
40–50	5	45	225	40
Total	40		990	

WORKED EXAMPLE **Solution**

Mean = Σfm / N = 990 / 40 = **24.75 marks**.

Median: N/2 = 20 → median class 20–30 (cf 28 first exceeds 20). L = 20, cf = 13, f = 15, h = 10.

Median = 20 + [(20 − 13)/15] × 10 = 20 + 4.67 = **24.67 marks**.

Mode: modal class 20–30 (highest f = 15). f_1 = 15, f_o = 8, f_2 = 7.

Mode = 20 + [(15 − 8)/(30 − 8 − 7)] × 10 = 20 + (7/15) × 10 = **24.67 marks**.

Check with empirical relation: 3(24.67) − 2(24.75) = 74.0 − 49.5 = 24.5 ≈ 24.67 ✓ (distribution almost symmetric).

WORKED EXAMPLE **Combined and weighted mean**

(a) Section A: 40 students, mean 60; Section B: 60 students, mean 70. Combined mean = (2,400 + 4,200)/100 = **66**.

(b) Weighted mean of marks 80 (weight 3), 70 (weight 2), 60 (weight 5) = (240 + 140 + 300)/10 = **68**.

FORMULA **GM and HM**

GM = $(X_1 \times X_2 \times \ldots \times X_n)^{(1/n)}$ = antilog (Σ log X / N)

HM = N / Σ(1/X)

For two positive numbers: AM ≥ GM ≥ HM, and GM^2 = AM × HM

- **GM** – used for averaging **rates of growth**, ratios, percentages, index numbers.
- **HM** – used for averaging **speeds, rates per unit** (e.g. km per hour when distance is constant).

WORKED EXAMPLE **GM and HM**

(a) A company's sales grew by 10%, 20% and 32% in three years. Average growth = GM of 1.10, 1.20, 1.32 = (1.7424)^(1/3) = 1.2033 → **20.33%** (not 20.67% given by AM).

(b) A car goes 60 km at 30 km/h and returns at 60 km/h. Average speed = HM = 2/(1/30 + 1/60) = **40 km/h**.

Basis	Mean	Median	Mode
Based on all items	Yes	No	No
Affected by extremes	Yes	No	No
Open-end classes	Not possible directly	Possible	Possible
Graphical location	No	Ogive	Histogram
Algebraic treatment	Yes	No	No
Best for	Normal data, further analysis	Skewed data (income, wealth), qualitative ranks	Most common size (fashion, shoes)

COMMON MISTAKE **In the median formula, use the cumulative frequency of the class before the median class, not of the median class itself.**

FORMULA Properties used in MCQs

Sum of deviations from the mean = 0; sum of squared deviations is least from the **mean**; sum of absolute deviations is least from the **median**.

Combined mean = $(n_1\bar{x}_1 + n_2\bar{x}_2) \div (n_1 + n_2)$. If $y = a + bx$ then $\bar{y} = a + b\,\bar{x}$, median and mode change the same way.

Empirical relation: Mode = 3 Median − 2 Mean. For two numbers: AM × HM = GM^2.

Quartiles: $Q_1 = (n + 1)/4$ th item (individual series); for grouped data $Q_1 = L + [(N/4 - cf)/f] \times h$. Deciles and percentiles similarly ($D_5 = P_{50}$ = median).

14.2 Dispersion

Dispersion measures the **extent of scatter or variation** of individual values around the central value. Two series may have the same mean but very different spreads.

Absolute measures (in original units)	Relative measures (coefficients, unit-free)
Range	Coefficient of range
Quartile deviation	Coefficient of QD
Mean deviation	Coefficient of MD
Standard deviation	Coefficient of SD; **coefficient of variation**

Relative measures are used to **compare** variability of series with different units or averages.

FORMULA Range

Range = L − S (largest − smallest); Coefficient of range = (L − S)/(L + S)

Simple but based only on two extreme values. Used in **quality control** (control charts), stock prices (52-week range), weather reports.

FORMULA Quartile deviation

Inter-quartile range = Q3 − Q1; QD = (Q3 − Q1)/2

Coefficient of QD = (Q3 − Q1)/(Q3 + Q1)

WORKED EXAMPLE Quartile deviation

Q1 = 20, Q3 = 40. QD = (40 − 20)/2 = **10**; coefficient = 20/60 = **0.33**.

Not affected by extremes; suitable for open-end distributions; ignores 50% of data.

FORMULA Mean deviation

MD = $\Sigma|X - A| / N$ (A = mean or median; MD is least when taken from the median)

Discrete/continuous: MD = $\Sigma f|X - A| / N$

Coefficient of MD = MD ÷ A (the average used)

WORKED EXAMPLE Mean deviation

Data: 2, 4, 6, 8, 10. Mean = 6. |deviations| = 4, 2, 0, 2, 4 → Σ = 12. MD = 12/5 = **2.4**; coefficient = 2.4/6 = **0.4**.

Based on all values but ignores signs (not suitable for algebraic treatment).

Standard deviation (introduced by **Karl Pearson**, 1893) is the **square root of the mean of the squared deviations from the arithmetic mean**. It is the best and most widely used measure of dispersion.

FORMULA Standard deviation

Individual series: $\sigma = \sqrt{[\Sigma(X - \bar{X})^2 / N]} = \sqrt{[\Sigma X^2/N - (\Sigma X/N)^2]}$

Shortcut (assumed mean): $\sigma = \sqrt{[\Sigma d^2/N - (\Sigma d/N)^2]}$

Discrete/continuous: $\sigma = \sqrt{[\Sigma fd^2/N - (\Sigma fd/N)^2]}$; step deviation: multiply by h

Variance = σ^2

Combined SD: $\sigma_{12} = \sqrt{[(N_1\sigma_1^2 + N_2\sigma_2^2 + N_1d_1^2 + N_2d_2^2)/(N_1 + N_2)]}$, where $d_1 = \bar{X}_1 - \bar{X}_{12}$, $d_2 = \bar{X}_2 - \bar{X}_{12}$

Coefficient of variation (CV) = (σ / X̄) × 100

Properties of SD

- SD is **independent of change of origin** (adding a constant does not change σ) but **not of scale** (multiplying values by k multiplies σ by |k|).
- SD of the first n natural numbers = $\sqrt{[(n^2 - 1)/12]}$.
- For a normal distribution: mean ± 1σ covers **68.27%**, ± 2σ covers **95.45%**, ± 3σ covers **99.73%** of items.
- Relation (approximate, for moderately skewed data): QD ≈ (2/3)σ, MD ≈ (4/5)σ.

WORKED EXAMPLE SD and CV – individual series

Data: 2, 4, 4, 4, 5, 5, 7, 9 (N = 8). Mean = 40/8 = 5.

$(X - 5)^2$: 9, 1, 1, 1, 0, 0, 4, 16 → Σ = 32. Variance = 32/8 = 4. **σ = 2**.

CV = 2/5 × 100 = **40%**.

WORKED EXAMPLE Which share is more stable? (CV comparison)

Share A: mean price ₹100, σ = ₹12. Share B: mean price ₹250, σ = ₹20.

CV(A) = 12%; CV(B) = 8%. Share **B** has lower CV → **more consistent/stable**, even though its SD is larger.

WORKED EXAMPLE SD of a frequency distribution (step deviation)

Classes 0–10, 10–20, 20–30, 30–40, 40–50 with f = 5, 8, 15, 7, 5. Take A = 25, h = 10: d' = −2, −1, 0, 1, 2.

fd' = −10, −8, 0, 7, 10 → Σfd' = −1. fd'^2 = 20, 8, 0, 7, 20 → Σfd'^2 = 55. N = 40.

$\sigma = 10 \times \sqrt{[55/40 - (-1/40)^2]} = 10 \times \sqrt{(1.375 - 0.000625)} = 10 \times \sqrt{1.374375} = 10 \times 1.1723 =$ **11.72 marks**.

Mean = 25 + (−1/40) × 10 = 24.75 (same as before ✓). CV = 11.72/24.75 × 100 = **47.4%**.

FORMULA Change of origin and scale; combined SD

If y = a + bx: SD(y) = |b| × SD(x); range, QD and MD also multiply by |b|; adding a constant changes none of them.

SD of first n natural numbers = $\sqrt{[(n^2 - 1)/12]}$. For two numbers a, b: SD = |a − b|/2.

Combined SD: $\sigma^2 = [n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)] \div (n_1 + n_2)$, where $d_i = \bar{x}_i$ – combined mean.

Normal relation (approx.): QD : MD : SD = 10 : 12 : 15. Coefficient of variation CV = σ/x̄ × 100.

Chapter 15: Probability

15.1 Basic terms and definitions

- **Random experiment**, **sample space** S, **event** (subset of S). **Mutually exclusive** – cannot occur together; **exhaustive** – together cover S; **equally likely**; **independent** – occurrence of one does not affect the other.
- **Classical definition:** P(A) = favourable cases ÷ total equally likely cases. $0 \le P(A) \le 1$; $P(A') = 1 - P(A)$.
- **Odds in favour** of A = m : n ⇒ P(A) = m/(m + n).

15.2 Theorems

Theorem	Formula
Addition	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$; for exclusive events P(A) + P(B)
Three events	$P(A \cup B \cup C) = \Sigma P(A) - \Sigma P(A \cap B) + P(A \cap B \cap C)$
Multiplication (independent)	$P(A \cap B) = P(A) \times P(B)$
Conditional	$P(A/B) = P(A \cap B) \div P(B)$
Compound (dependent)	$P(A \cap B) = P(A) \times P(B/A)$
Bayes' theorem	$P(A_i/B) = P(A_i)P(B/A_i) \div \Sigma P(A_j)P(B/A_j)$

15.3 Random variable and expectation

$E(X) = \Sigma x \cdot p(x)$; $E(X^2) = \Sigma x^2 \cdot p(x)$; **$Var(X) = E(X^2) - [E(X)]^2$**; $E(a + bX) = a + bE(X)$; $Var(a + bX) = b^2 Var(X)$.

WORKED EXAMPLE **Probability**

(a) Two dice: P(sum = 8) = 5/36; P(doublet) = 6/36 = 1/6.

(b) A card from a pack of 52: P(king or heart) = 4/52 + 13/52 − 1/52 = **4/13**.

(c) Bag with 5 red, 3 white: two drawn without replacement – P(both red) = ${}^5C_2/{}^8C_2$ = 10/28 = **5/14**.

(d) P(A) = 0.4, P(B) = 0.5, independent → P(at least one) = 1 − 0.6 × 0.5 = **0.7**.

(e) X takes 1, 2, 3 with probabilities 0.2, 0.5, 0.3 → E(X) = 0.2 + 1.0 + 0.9 = **2.1**; $E(X^2)$ = 0.2 + 2 + 2.7 = 4.9; Var = 4.9 − 4.41 = **0.49**.

(f) Leap year with 53 Sundays: 366 days = 52 weeks + 2 days → 7 equally likely pairs, 2 contain Sunday → **2/7**.

Chapter 16: Theoretical Distributions

16.1 Binomial distribution

$P(X = x) = {}^nC_x\, p^x\, q^{n-x}$, $x = 0, 1, \ldots, n$; $q = 1 - p$. **Mean = np; variance = npq** (variance < mean); mode – the integer part of $(n + 1)p$ (bimodal if $(n + 1)p$ is an integer). Symmetric when $p = q = 1/2$.

16.2 Poisson distribution

P(X = x) = e^(−m) $m^x \div x!$, x = 0, 1, 2 … ; **mean = variance = m**. Limiting form of binomial when n is large, p small, np = m finite. Used for rare events (accidents, misprints, defects).

16.3 Normal distribution

Bell-shaped, **symmetric**, mean = median = mode; total area 1; asymptotic to the x-axis; points of inflexion at μ ± σ. **$Z = (X - \mu)/\sigma$** gives the standard normal N(0, 1).

Range	Area
μ ± 1σ	68.27%
μ ± 2σ	95.45%
μ ± 3σ	99.73%
Quartile deviation	≈ 0.6745σ (about 2/3 σ)
Mean deviation	≈ 0.7979σ (about 4/5 σ)

WORKED EXAMPLE **Distributions**

(a) Binomial: n = 6, p = 1/2 – P(exactly 2 heads) = ${}^6C_2/64$ = **15/64**; mean 3, variance 1.5.

(b) If mean = 4 and variance = 3 for a binomial, q = 3/4, p = 1/4, n = 16.

(c) Poisson, m = 2: $P(0) = e^{-2} = 0.1353$; P(1) = 0.2707; P(at least 1) = **0.8647**.

(d) Normal μ = 60, σ = 5: P(X > 70) = P(Z > 2) = 0.5 − 0.4772 = **0.0228**.

Chapter 17: Correlation and Regression

Correlation is a statistical technique to measure the **degree and direction of relationship** between two variables.

Type	Meaning	Example
Positive	Both move in the same direction	Height and weight; advertisement and sales
Negative	Move in opposite directions	Price and quantity demanded
Linear / non-linear	Constant / changing ratio of change	–
Simple / multiple / partial	Two variables / three or more / two with others held constant	–

Methods of studying correlation

1. **Scatter diagram** – visual; shows direction and rough degree.
2. **Karl Pearson's coefficient of correlation (r)** – product-moment method.
3. **Spearman's rank correlation coefficient (R)** – for ranked/qualitative data.
4. Concurrent deviation method.

√x FORMULA Karl Pearson's coefficient of correlation

$r = \Sigma xy / \sqrt{(\Sigma x^2 \times \Sigma y^2)}$, where $x = X - \bar{X}$, $y = Y - \bar{Y}$

$r = [N\Sigma XY - \Sigma X \Sigma Y] / \sqrt{\{[N\Sigma X^2 - (\Sigma X)^2][N\Sigma Y^2 - (\Sigma Y)^2]\}}$

$r = \text{Cov}(X,Y) / (\sigma x\, \sigma y)$; $-1 \le r \le +1$

Probable error $PE = 0.6745\,(1 - r^2)/\sqrt{N}$; r is significant if $r > 6\,PE$

Coefficient of determination $= r^2$

Value of r	Interpretation
+1	Perfect positive correlation
+0.75 to +1	High positive
+0.25 to +0.75	Moderate positive
0	No (linear) correlation
−1	Perfect negative

Properties of r

Lies between −1 and +1; independent of change of origin and scale; symmetric (rxy = ryx); r is the geometric mean of the two regression coefficients.

WORKED EXAMPLE Karl Pearson's r

X: 1, 2, 3, 4, 5; Y: 2, 4, 5, 4, 5. $\bar{X} = 3$, $\bar{Y} = 4$.

x: −2, −1, 0, 1, 2; y: −2, 0, 1, 0, 1. xy: 4, 0, 0, 0, 2 → $\Sigma xy = 6$. $\Sigma x^2 = 10$; $\Sigma y^2 = 4 + 0 + 1 + 0 + 1 = 6$.

$r = 6 / \sqrt{(10 \times 6)} = 6 / 7.746 =$ **0.775** → high positive correlation.

Spearman's rank correlation

√x FORMULA Rank correlation

$R = 1 - [6\Sigma d^2 / (N(N^2 - 1))]$

With tied ranks: add $(m^3 - m)/12$ to Σd^2 for each tie

WORKED EXAMPLE Rank correlation

Two judges rank 5 contestants; $\Sigma d^2 = 6$. $R = 1 - (6 \times 6)/(5 \times 24) = 1 - 36/120 =$ **0.70**.

Correlation vs causation

High correlation does not prove cause and effect – it may be due to chance (**spurious / nonsense correlation**), a third common factor, or mutual influence.

Regression is the estimation of the unknown value of one variable from the known value of another, using the average relationship between them. The term was introduced by **Sir Francis Galton**.

FORMULA Regression equations

Y on X: $Y - \bar{Y} = byx\ (X - \bar{X})$, $byx = \Sigma xy / \Sigma x^2 = r\ (\sigma y/\sigma x)$

X on Y: $X - \bar{X} = bxy\ (Y - \bar{Y})$, $bxy = \Sigma xy / \Sigma y^2 = r\ (\sigma x/\sigma y)$

$r = \pm\sqrt{(byx \times bxy)}$ (sign same as the regression coefficients)

Both regression lines pass through $(\bar{X}, \bar{Y})$

WORKED EXAMPLE Regression equations (same data as above)

$byx = 6/10 = 0.6 \rightarrow Y - 4 = 0.6\ (X - 3) \rightarrow$ **$Y = 2.2 + 0.6X$**.

$bxy = 6/6 = 1 \rightarrow X - 3 = 1\ (Y - 4) \rightarrow$ **$X = Y - 1$**.

$r = \sqrt{(0.6 \times 1)} = 0.775$ ✓. Estimate Y when X = 6: Y = 2.2 + 3.6 = **5.8**.

Correlation vs regression

Basis	Correlation	Regression
Purpose	Degree and direction of relationship	Estimating one variable from another
Variables	Both random; no dependent/independent distinction	Dependent and independent variables
Coefficient	rxy = ryx (symmetric)	byx ≠ bxy
Range	−1 to +1	Can exceed 1 (but both coefficients cannot exceed 1 together)
Units	Pure number	In units of the dependent variable
Change of scale	Independent of origin and scale	Independent of origin, not of scale

FORMULA Relations ICAI tests every time

$r = \pm\sqrt{(b_{yx} \times b_{xy})}$ (same sign as the regression coefficients); both regression lines pass through $(\bar{x}, \bar{y})$.

Change of origin and scale: r is unaffected in magnitude (sign changes if one scale factor is negative); b_{yx} changes by the factor (scale of y ÷ scale of x).

Coefficient of determination = r^2; coefficient of non-determination = $1 - r^2$.

Spearman's rank correlation: $R = 1 - 6\Sigma d^2 \div [n(n^2 - 1)]$; with ties add $(m^3 - m)/12$ to Σd^2 for each tie.

Probable error = $0.6745\ (1 - r^2)/\sqrt{n}$.

Chapter 18: Index Numbers

An **index number** is a statistical measure of **relative change** in a variable or group of related variables over time or space, expressed as a percentage of a **base** (base = 100). Called "economic barometers".

Uses

Measuring inflation (CPI, WPI), deflating values (real income), wage negotiation and DA, business forecasting, policy formulation, stock market trends (**Sensex, Nifty**).

Methods

Method	Formula
Simple aggregative	$P_{01} = \Sigma p_1 / \Sigma p_0 \times 100$
Simple average of relatives	$P_{01} = \Sigma(p_1/p_0 \times 100) / N$
Laspeyres (base-year quantities)	$P_{01} = \Sigma p_1 q_0 / \Sigma p_0 q_0 \times 100$
Paasche (current-year quantities)	$P_{01} = \Sigma p_1 q_1 / \Sigma p_0 q_1 \times 100$
Fisher's ideal	$P_{01} = \sqrt{(\text{Laspeyres} \times \text{Paasche})}$
Dorbish–Bowley	$(L + P)/2$
Marshall–Edgeworth	$\Sigma p_1(q_0 + q_1) / \Sigma p_0(q_0 + q_1) \times 100$
Kelly (fixed weights)	$\Sigma p_1 q / \Sigma p_0 q \times 100$

WORKED EXAMPLE **Laspeyres, Paasche and Fisher**

Commodity A: $p_0 = 10$, $q_0 = 5$, $p_1 = 12$, $q_1 = 6$. Commodity B: $p_0 = 8$, $q_0 = 10$, $p_1 = 10$, $q_1 = 8$.

$\Sigma p_1 q_0 = 60 + 100 = 160$; $\Sigma p_0 q_0 = 50 + 80 = 130$; $\Sigma p_1 q_1 = 72 + 80 = 152$; $\Sigma p_0 q_1 = 60 + 64 = 124$.

Laspeyres = 160/130 × 100 = **123.08**. Paasche = 152/124 × 100 = **122.58**.

Fisher = $\sqrt{(123.08 \times 122.58)} = \sqrt{15{,}087}$ = **122.83**.

Tests of adequacy

- **Time reversal test**: $P_{01} \times P_{10} = 1$.
- **Factor reversal test**: $P_{01} \times Q_{01} = \Sigma p_1 q_1 / \Sigma p_0 q_0$.
- **Fisher's index satisfies both** tests – hence called **ideal**. Laspeyres and Paasche satisfy neither.

Consumer price index (cost of living index)

FORMULA **CPI methods**

Aggregate expenditure method: CPI = $\Sigma p_1 q_0 / \Sigma p_0 q_0 \times 100$

Family budget method: CPI = $\Sigma PW / \Sigma W$, where P = price relatives, W = $p_0 q_0$ (base-year expenditure weights)

Real wage = Money wage ÷ CPI × 100; Purchasing power of money = 100 ÷ CPI

WORKED EXAMPLE **Real wage**

A worker's wage rose from ₹20,000 (2020) to ₹26,000 (2025); CPI rose from 100 to 125. Real wage in 2025 = 26,000 ÷ 125 × 100 = **₹20,800** → real increase only 4%, though money wage rose 30%.

Chain base index and base shifting

- **Fixed base** – all years compared with one base year. **Chain base** – each year compared with the previous year (link relatives), then chained.
- **Base shifting**: new index = old index ÷ old index of new base year × 100.
- **Splicing** connects two overlapping series.

√× FORMULA Tests and other points

Fisher's ideal index = √(Laspeyres × Paasche); it satisfies both the **time reversal** ($P_{01} \times P_{10} = 1$) and **factor reversal** ($P_{01} \times Q_{01} = \Sigma p_1q_1/\Sigma p_oq_o$) tests.

Laspeyres overstates and Paasche understates (usually). Marshall–Edgeworth uses ($q_o + q_1$) weights.

Chain base index: link relative = current price ÷ previous year price × 100; chain index = previous chain index × link relative ÷ 100.

Deflating: real value = money value ÷ price index × 100. **Splicing** joins two index series with different bases.