Skillshort

Self-Study Preparation Pack

CMA FOUNDATION

Paper 3: Fundamentals of Business Mathematics and Statistics (FBMS)

Module-wise Detailed Notes

For December 2026 and June 2027 terms · ICMAI Syllabus 2022 · Section A Mathematics 40% · Section B Statistics 60%

50 MCQs × 2 marks | 1 hour (Papers 3 + 4 in one 2-hour session) | No negative marking | Calculator allowed VERIFY

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This subject pack has 6 parts

1 Chapter-wise notes

2 Important topics & weightage

3 Past papers: trends & links

4 Mock tests + solutions

5 Chapter practice questions

6 Last-week plan & strategy

Prepared by Skillshort for self-study. Original explanations and questions; not an official ICAI/ICMAI/ICSI publication. Items marked VERIFY should be checked against the latest official material.

How to use these notes

Paper 3 is almost entirely **short numericals**: 20 questions on business mathematics and 30 on statistics, about 70 seconds each. Every module below gives the formulas, the standard question types and solved one-step examples. Learn a formula, then immediately do that module's set in file 05.

File	Use it for
01 Module Notes (this file)	Formulas, concepts and solved examples
02 Important Topics & Weightage	Official weightage vs the June 2026 paper
03 Past Papers – Trend Analysis & Links	ICMAI question patterns and official links
04 Mock Tests 1–3 with Solutions	Three full 50-MCQ papers with working
05 Module-wise Practice Questions	10 MCQs per module with working
06 Last-Week Plan & Exam Strategy	Formula sheet, time plan, exam-hall plan

Paper at a glance

Item	Detail
Marks / questions	100 marks · 50 MCQs × 2 marks · **no negative marking** (Q1–50 of the afternoon OMR booklet)
Time	1 hour per paper; Papers 3 and 4 share the 2:00–4:00 pm session VERIFY
Section A – Business Mathematics (40%)	Arithmetic 15% · Algebra 20% · Calculus 5%
Section B – Statistics (60%)	Statistical representation of data 5% · Measures of central tendency and dispersion 15% · Correlation and regression 15% · Probability 15% · Index numbers and time series 10%

STRATEGY Calculator

Practise the memory keys (M+, M−, MR) and powers: for 1.08^5 press 1.08 × = four times on most calculators. Check the admit card for the calculator rule VERIFY.

Module 1: Arithmetic

1.1 Ratio, proportion and variation

- **Ratio** a : b = a/b compares two quantities of the same kind.
- **Compound ratio** of a:b and c:d = ac : bd. **Duplicate ratio** of a:b = $a^2 : b^2$. **Sub-duplicate** = √a : √b. **Triplicate** = $a^3 : b^3$.
- **Inverse ratio** of a : b = b : a.
- **Proportion**: a : b = c : d ⇔ ad = bc (product of extremes = product of means).
- **Continued proportion**: a : b = b : c ⇒ b^2 = ac (b is the **mean proportional**). Third proportional to a and b = b^2/a. Fourth proportional to a, b, c = bc/a.

Properties of proportion (if a/b = c/d)

Property	Result
Invertendo	b/a = d/c
Alternendo	a/c = b/d
Componendo	(a + b)/b = (c + d)/d
Dividendo	(a − b)/b = (c − d)/d
Componendo and dividendo	(a + b)/(a − b) = (c + d)/(c − d)

WORKED EXAMPLE Ratio problems

(a) Divide ₹72,000 among A, B, C in the ratio 2 : 3 : 4. Total parts 9 → A ₹16,000, B ₹24,000, C ₹32,000.

(b) Mean proportional between 4 and 25 = √100 = **10**.

(c) Two numbers are in ratio 3 : 5; if 9 is added to each, the ratio becomes 3 : 4. Let numbers 3x and 5x: (3x+9)/(5x+9) = 3/4 → 12x + 36 = 15x + 27 → x = 3 → numbers **9 and 15**.

(d) Partners A and B invest ₹40,000 for 12 months and ₹60,000 for 6 months. Profit ratio = 4,80,000 : 3,60,000 = **4 : 3**.

Variation

- **Direct variation**: y ∝ x ⇒ y = kx. **Inverse variation**: y ∝ 1/x ⇒ xy = k. **Joint variation**: z ∝ xy.

WORKED EXAMPLE Variation

(a) x varies directly as y; x = 12 when y = 4. When y = 7: k = 3 → x = **21**.

(b) x varies inversely as y; x = 6 when y = 10 (xy = 60). When y = 15: x = **4**.

(c) z varies jointly as x and y; z = 24 when x = 2, y = 3 (k = 4). When x = 5, y = 2: z = **40**.

(d) 12 workers finish a job in 15 days. 20 workers (inverse variation) take 12 × 15 ÷ 20 = **9 days**.

WORKED EXAMPLE Time and work

A can do a job in 10 days and B in 15 days: 1/10 + 1/15 = 1/6 → together **6 days**. Wages are shared in the ratio of work done, 3 : 2.

1.2 Simple interest

Interest is the payment for the use of borrowed money. The original sum is the **principal (P)**; principal + interest is the **amount (A)**.

FORMULA Simple interest

SI = P × R × T ÷ 100

A = P + SI = P (1 + RT/100)

P = 100 × SI ÷ (R × T); R = 100 × SI ÷ (P × T); T = 100 × SI ÷ (P × R)

WORKED EXAMPLE Simple interest

(a) Find SI on ₹50,000 at 8% p.a. for 3 years. SI = 50,000 × 8 × 3 ÷ 100 = **₹12,000**; Amount = ₹62,000.

(b) In how many years will a sum double at 10% simple interest? SI = P → P = P × 10 × T/100 → **T = 10 years**.

(c) A sum amounts to ₹13,200 in 2 years and ₹14,400 in 4 years at SI. Interest for 2 years = ₹1,200 → per year ₹600. Principal = 13,200 − 1,200 = ₹12,000; R = 600 × 100 ÷ 12,000 = **5%**.

1.3 Compound interest and present value

Under **compound interest (CI)**, interest is added to the principal at the end of each period, and interest for the next period is calculated on the increased amount.

FORMULA Compound interest

A = P (1 + r/100)^n ; CI = A − P

Compounded k times a year: A = P (1 + r/(100k))^(nk)

Effective rate of interest: E = (1 + r/(100k))^k − 1

Depreciation (reducing balance): Value after n years = P (1 − r/100)^n

Population growth: P_n = P_0 (1 + r/100)^n

WORKED EXAMPLE Compound interest

(a) ₹1,00,000 at 10% p.a. compounded annually for 2 years: A = 1,00,000 × $(1.1)^2$ = ₹1,21,000; **CI = ₹21,000** (SI would be ₹20,000; difference ₹1,000 = $P(r/100)^2$).

(b) Compounded half-yearly: A = 1,00,000 × $(1.05)^4$ = ₹1,21,550.63; CI = ₹21,550.63.

(c) Effective rate of 12% p.a. compounded monthly: $(1.01)^{12}$ − 1 = 0.1268 → **12.68%**.

(d) A machine costing ₹2,00,000 depreciates at 10% p.a. (reducing balance). Value after 3 years = 2,00,000 × $(0.9)^3$ = **₹1,45,800**.

MEMORY TRICK Difference between CI and SI for 2 years = P (r/100)². For 3 years = P (r/100)² (3 + r/100).

Present value and discounting

FORMULA Present value

PV = A ÷ (1 + r/100)^n

WORKED EXAMPLE Present value

What sum invested now at 8% p.a. compound will become ₹1,00,000 in 3 years? PV = 1,00,000 ÷ $(1.08)^3$ = 1,00,000 ÷ 1.259712 = **₹79,383** (approx).

1.4 Annuities

An **annuity** is a series of **equal payments** made at **equal intervals** of time (EMIs, insurance premiums, pensions, SIP instalments, rent).

Type	Meaning
Annuity regular (ordinary/immediate)	Payment at the **end** of each period
Annuity due	Payment at the **beginning** of each period
Perpetuity	Payments continue forever
Deferred annuity	Payments start after a certain number of periods
Contingent annuity	Depends on an uncertain event (life annuity)

FORMULA Annuity formulas (i = rate per period as a decimal, n = number of periods)

Future value of annuity regular: FV = A × [(1 + i)^n − 1] ÷ i

Present value of annuity regular: PV = A × [1 − (1 + i)^−n] ÷ i

Annuity due: FV(due) = FV(regular) × (1 + i); PV(due) = PV(regular) × (1 + i)

Perpetuity: PV = A ÷ i

EMI on a loan: EMI = P × i × (1 + i)^n ÷ [(1 + i)^n − 1]

Sinking fund instalment: A = FV × i ÷ [(1 + i)^n − 1]

WORKED EXAMPLE Annuities

(a) ₹10,000 deposited at the end of each year for 5 years at 10%: FV = 10,000 × $(1.1^5 - 1)/0.1$ = 10,000 × 6.1051 = **₹61,051**.

(b) PV of ₹10,000 received at the end of each year for 5 years at 10%: 10,000 × $(1 - 1.1^{-5})/0.1$ = 10,000 × 3.7908 = **₹37,908**.

(c) If the payments are at the beginning of each year (annuity due): FV = 61,051 × 1.1 = ₹67,156.

(d) EMI on a loan of ₹5,00,000 at 12% p.a. for 2 years (monthly): i = 0.01, n = 24; $(1.01)^{24}$ = 1.26973. EMI = 5,00,000 × 0.01 × 1.26973 ÷ 0.26973 = **₹23,537** (approx).

(e) Perpetuity of ₹6,000 a year at 8%: PV = 6,000 ÷ 0.08 = **₹75,000**.

1.5 Arithmetic and geometric progressions

a, a + d, a + 2d ... ; $\mathbf{t_n = a + (n - 1)d}$; $\mathbf{S_n = n/2\ [2a + (n - 1)d] = n/2\ (a + l)}$. Arithmetic mean of a and b = (a + b)/2. Three numbers in AP: a − d, a, a + d.

a, ar, ar^2 ... ; $\mathbf{t_n = ar^{n-1}}$; $\mathbf{S_n = a(r^n - 1)/(r - 1)}$ (r ≠ 1); infinite GP (|r| < 1): $\mathbf{S\infty = a/(1 - r)}$. Geometric mean of a, b = √(ab). Three numbers in GP: a/r, a, ar.

- For positive a, b: **AM ≥ GM ≥ HM** and GM^2 = AM × HM.

Series	Sum
1 + 2 + ... + n	n(n + 1)/2
$1^2 + 2^2 + ... + n^2$	n(n + 1)(2n + 1)/6
$1^3 + 2^3 + ... + n^3$	$[n(n + 1)/2]^2$
Sum of first n odd numbers	n^2
0.3 + 0.33 + 0.333 + ... n terms	$(1/3)[n - (1/9)(1 - 10^{-n})]$

WORKED EXAMPLE Sequences

(a) 15th term of 3, 7, 11 ... = 3 + 14 × 4 = **59**; sum of 15 terms = 15/2 (3 + 59) = **465**.

(b) Sum of all two-digit numbers divisible by 3: 12, 15 ... 99 → n = 30 → S = 30/2 (12 + 99) = **1,665**.

(c) 8th term of 2, 6, 18 ... = 2 × 3^7 = **4,374**. S∞ of 1 + 1/2 + 1/4 + ... = **2**.

(d) Business: salary ₹30,000 rising ₹2,000 a year – total earnings in 10 years = 10/2 [60,000 + 9 × 2,000] = **₹3,90,000**.

1.6 Time and distance

Rule	Formula
Basic	Distance = speed × time
Units	km/h → m/s: × 5/18; m/s → km/h: × 18/5
Average speed, equal distances at x and y	**2xy ÷ (x + y)** (not the simple average)
Average speed, equal times at x and y	(x + y) ÷ 2
Relative speed	Same direction: difference of speeds; opposite: sum
Train passing a pole / a platform	Time = train length ÷ speed; (train + platform length) ÷ speed
Boats (speed u in still water, stream v)	Downstream u + v; upstream u − v

WORKED EXAMPLE Time and distance

(a) A car goes from A to B at 40 km/h and returns at 60 km/h. Average speed = 2 × 40 × 60 ÷ 100 = **48 km/h**.

(b) A 200 m train at 72 km/h (= 20 m/s) passes a pole in **10 seconds** and a 300 m platform in 500 ÷ 20 = **25 seconds**.

(c) Two trains 300 km apart move towards each other at 70 and 80 km/h: they meet after 300 ÷ 150 = **2 hours**.

Module 2: Algebra

2.1 Set theory and Venn diagrams

A **set** is a well-defined collection of distinct objects. Notation: A = {1, 2, 3} (roster form) or A = {x : x is a natural number less than 4} (set-builder form).

Term	Meaning
Empty (null) set ⌀	No element
Singleton	One element
Finite / infinite	Countable number of elements / not
Subset A ⊆ B	Every element of A is in B; number of subsets of a set with n elements = 2^n
Proper subset	A ⊂ B and A ≠ B
Power set P(A)	Set of all subsets of A
Universal set U	Set containing all elements under consideration
Union A ∪ B	Elements in A or B or both
Intersection A ∩ B	Elements in both
Difference A − B	Elements in A but not in B
Complement A'	Elements in U but not in A
Disjoint sets	A ∩ B = ⌀

FORMULA Set laws and counting

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

De Morgan's laws: $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$

WORKED EXAMPLE Set application

In a class of 100 students, 60 study Accounts, 50 study Economics and 20 study both.

At least one = 60 + 50 − 20 = **90**. Neither = 100 − 90 = **10**. Only Accounts = 60 − 20 = **40**; only Economics = **30**.

EXAM TIP Draw a Venn diagram for every set problem – fill the intersection first, then the "only" regions.

WORKED EXAMPLE Venn diagram percentages

In a survey, 60% like tea, 50% like coffee and 20% like both. Liking at least one = 60 + 50 − 20 = 90% → **neither = 10%**; only tea = 40%; only coffee = 30%.

For three sets: $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$.

2.2 Indices

Law	Result
Product	$a^m \times a^n = a^{m+n}$
Quotient	$a^m \div a^n = a^{m-n}$
Power	$(a^m)^n = a^{mn}$
Zero index	$a^0 = 1$ ($a \neq 0$)
Negative index	$a^{-n} = 1/a^n$
Fractional index	a^(1/n) = $\sqrt[n]{a}$; a^(m/n) = $(\sqrt[n]{a})^m$

Law	Result
Product of bases	$(ab)^n = a^n b^n$; $(a/b)^n = a^n/b^n$
Equal powers	If $a^x = a^y$ ($a \neq 1$) then $x = y$

WORKED EXAMPLE Indices

(a) Simplify (27)^(2/3) × (16)^(−1/4) = 9 × 1/2 = **4.5**.

(b) If $2^x = 3^y = 6^{(-z)}$, then 1/x + 1/y + 1/z = **0** (let each = k: 2 = k^(1/x), 3 = k^(1/y), 6 = k^(−1/z); 2 × 3 = 6 → 1/x + 1/y = −1/z).

(c) If x = 3^(1/3) + 3^(−1/3), then $3x^3 - 9x$ = **10** (cube: $x^3 = 3 + 1/3 + 3x \rightarrow 3x^3 = 10 + 9x$).

2.3 Logarithms

If $a^x = N$ ($a > 0$, $a \neq 1$, $N > 0$), then **$x = \log_a N$**. Common logs have base 10; natural logs base e.

Rule	Result
Product	log (mn) = log m + log n
Quotient	log (m/n) = log m − log n
Power	$\log m^n = n \log m$
Base change	$\log_a N = \log_b N \div \log_b a$; $\log_a b \times \log_b a = 1$
Special values	$\log_a a = 1$; $\log_a 1 = 0$; $a^{(\log_a N)} = N$
Power of base	$\log_{(a^k)} N = (1/k) \log_a N$

WORKED EXAMPLE Logarithms

(a) $\log_2 64 = 6$ ($2^6 = 64$). log √2 of 8 = $\log_2 8 \div (1/2)$ = **6**.

(b) Solve $\log_x 4 + \log_x 16 + \log_x 64 = 12 \rightarrow \log_x (4 \times 16 \times 64) = \log_x 4096 = 12 \rightarrow x^{12} = 4096 = 2^{12} \rightarrow$ **x = 2**.

(c) log 2 = 0.3010, log 3 = 0.4771 → log 72 = 3 log 2 + 2 log 3 = 0.9030 + 0.9542 = **1.8572**.

(d) Number of digits in 2^{50}: 50 × 0.3010 = 15.05 → characteristic 15 → **16 digits**.

(e) Antilog: if log x = 2.4771, x = antilog 2.4771 = 10^2 × 10^0.4771 = **300**.

MEMORY TRICK Characteristic and mantissa

For a number > 1, characteristic = (number of digits before the decimal point) − 1. For 0.00345, characteristic = −3 (written $\bar{3}$). Mantissa is always positive and taken from tables.

2.4 Permutations and combinations

- **Multiplication rule:** if one job can be done in m ways and another in n ways, both together in **m × n** ways. **Addition rule:** either of two exclusive jobs in **m + n** ways.
- n! = n × (n − 1) × ... × 1; 0! = 1.

Situation	Formula
r objects out of n distinct	${}^nP_r = n! \div (n - r)!$
All n distinct	n!
With repetition allowed (r places)	n^r
n objects with p alike, q alike...	n! ÷ (p! q! ...)
Circular (n distinct)	(n − 1)!
Circular where clockwise = anticlockwise (necklace, garland)	(n − 1)! ÷ 2
Two particular objects always together	(n − 1)! × 2!
Two particular objects never together	n! − (n − 1)! × 2!

${}^nC_r = n! \div [r!\,(n-r)!]$; ${}^nC_r = {}^nC_{n-r}$; ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$; ${}^nP_r = {}^nC_r \times r!$; total selections of at least one from $n = 2^n - 1$.

Situation	Formula
Diagonals of an n-sided polygon	${}^nC_2 - n$
Lines through n points (no 3 collinear)	${}^nC_2$
Triangles from n points (m collinear)	${}^nC_3 - {}^mC_3$
Handshakes among n persons	${}^nC_2$
A particular object always included	${}^{n-1}C_{r-1}$; always excluded ${}^{n-1}C_r$

WORKED EXAMPLE P & C

(a) Arrangements of the letters of "ACCOUNTANT": 10 letters – A 2, C 2, N 2, T 2, O 1, U 1 → $10! \div (2!)^4 = 3{,}628{,}800 \div 16 =$ **2,26,800**.

(b) 7 people around a round table: 6! = **720**; two particular people together: 5! × 2 = **240**.

(c) Committee of 5 from 6 men and 4 women with at least 2 women: ${}^4C_2 \cdot {}^6C_3 + {}^4C_3 \cdot {}^6C_2 + {}^4C_4 \cdot {}^6C_1 = 120 + 60 + 6 =$ **186**.

(d) ${}^nP_r = r! \times {}^nC_r$, so ${}^nP_3 = 6 \times {}^nC_3$ for every n. If ${}^nC_2 = 45$, $n(n-1) = 90 \rightarrow$ **n = 10**.

(e) 4-digit numbers from 1–9 without repetition: ${}^9P_4 =$ **3,024**; with repetition: $9^4 = 6{,}561$.

2.5 Quadratic equations

$ax^2 + bx + c = 0$ $(a \neq 0)$. Roots: $\mathbf{x = [-b \pm \sqrt{(b^2 - 4ac)}] \div 2a}$.

Item	Result
Sum of roots α + β	−b/a
Product αβ	c/a
Discriminant $D = b^2 - 4ac$	D > 0 real & unequal; D = 0 real & equal; D < 0 imaginary; D a perfect square (a, b, c rational) → rational roots
Equation from roots	$x^2 - (\alpha + \beta)x + \alpha\beta = 0$
Useful identities	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$; $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$; $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
Irrational roots	Occur in conjugate pairs $(p + \sqrt{q}, p - \sqrt{q})$

WORKED EXAMPLE Quadratics

(a) $x^2 - x - 6 = 0 \rightarrow \alpha + \beta = 1$, $\alpha\beta = -6 \rightarrow \alpha^2 + \beta^2 = 1 + 12 = 13$; $\alpha^3 + \beta^3 = 1 + 18 = 19$.

(b) Roots of $2x^2 - 6x + 3 = 0$ are α, β. Equation with roots α/β and β/α: sum $= (\alpha^2 + \beta^2)/\alpha\beta = (9 - 3)/1.5 = 4$; product $= 1 \rightarrow$ $\mathbf{x^2 - 4x + 1 = 0}$.

(c) If one root of $x^2 - 5x + k = 0$ is 2, then $4 - 10 + k = 0 \rightarrow k = 6$; other root 3.

Module 3: Calculus

3.1 Functions

- A **function** f: A → B assigns to each element of A **exactly one** element of B. Types: **one-one (injective)**, **onto (surjective)** – range = co-domain, **bijective** (both), constant, identity.
- Composite: (f ∘ g)(x) = f(g(x)). Inverse exists only for bijections. Number of functions from A (m) to B (n) = n^m.

WORKED EXAMPLE **f(x) = 2x + 3, g(x) = x² → f(g(2)) = f(4) = 11; g(f(2)) = g(7) = 49. Inverse of f: y = 2x + 3 → $f^{-1}(x) = (x - 3)/2$.**

- **Business functions:** demand p = f(x); revenue R(x) = p·x; cost C(x) = fixed cost + variable cost; profit P(x) = R(x) – C(x); break-even where R = C.

3.2 Differentiation – rules

Function	Derivative
x^n	$n\,x^{n-1}$
constant	0
e^x ; a^x	e^x ; $a^x \log_e a$
log x	1/x
u ± v	u′ ± v′
u·v (product)	u v′ + v u′
u/v (quotient)	(v u′ – u v′)/v²
f(g(x)) (chain)	f′(g(x)) · g′(x)

- **Second derivative** d^2y/dx^2; **implicit** and **parametric** differentiation: dy/dx = (dy/dt)/(dx/dt).

3.3 Revenue and cost functions; optimisation

- **Marginal cost** MC = dC/dx; **marginal revenue** MR = dR/dx; average cost AC = C/x.
- **Maxima/minima:** set f′(x) = 0; if f″(x) < 0 → maximum; f″(x) > 0 → minimum. Profit is maximum where MR = MC and slope condition holds.
- **Elasticity of demand** = −(p/x)(dx/dp).

WORKED EXAMPLE **Differential calculus**

(a) C(x) = 0.5x² + 20x + 500 → MC = x + 20; at x = 30, **MC = ₹50**.

(b) R(x) = 100x – 2x² → MR = 100 − 4x; R maximum when 100 − 4x = 0 → x = 25; R″ = −4 < 0 ✔ → max revenue **₹1,250**.

(c) y = x³ – 6x² + 9x → y′ = 3x² – 12x + 9 = 0 → x = 1 or 3; y″ = 6x – 12: at x = 1, −6 (max, y = 4); at x = 3, +6 (min, y = 0).

WORKED EXAMPLE **Minimum average cost**

C(x) = 2x² + 5x + 200 → AC = 2x + 5 + 200/x. d(AC)/dx = 2 − 200/x² = 0 → **x = 10**; second derivative 400/x³ > 0 → minimum. Minimum AC = 20 + 5 + 20 = **₹45**. At the minimum, AC = MC (MC = 4x + 5 = 45 ✔).

Module 4: Statistical Representation of Data

4.1 Meaning, data and collection

Statistics in the plural sense means numerical facts (data); in the singular sense it is the **science of collecting, organising, presenting, analysing and interpreting** numerical data. Croxton and Cowden: "Statistics may be defined as the collection, presentation, analysis and interpretation of numerical data."

Functions

Simplifies complex data, facilitates comparison, helps in forecasting and policy-making, tests hypotheses, and enlarges individual knowledge.

Limitations

Studies only aggregates (not individuals), deals only with quantitative data, results are true only on average, can be misused, and requires expertise.

Uses in business

Market research, quality control, sales forecasting, production planning, financial analysis, inventory control, HR (wage analysis), and insurance (actuarial).

- **Variable** – a characteristic that can be measured (height, wages): **discrete** (whole numbers) or **continuous** (any value in a range). **Attribute** – a qualitative characteristic that cannot be measured, only counted (honesty, literacy, colour).
- **Primary data** are collected first-hand for the purpose; **secondary data** were collected by someone else (published reports, websites).

4.2 Classification, tabulation and frequency distribution

Classification is the process of arranging data into groups or classes according to common characteristics – **geographical** (by place), **chronological** (by time), **qualitative** (by attributes – sex, literacy), **quantitative** (by measurable characteristics – income, marks).

Frequency distribution – terms

Term	Meaning
Class interval	Group such as 10–20
Class limits	Lower (10) and upper (20) limits
Class width (magnitude)	Upper limit – lower limit
Mid-value (class mark)	(Lower + upper limit) ÷ 2
Frequency	Number of observations in a class
Exclusive method	10–20, 20–30 (upper limit excluded)
Inclusive method	10–19, 20–29 (both limits included)
Open-end class	"Below 10" or "60 and above"
Cumulative frequency	Running total of frequencies (less than / more than type)
Relative frequency	Class frequency ÷ total frequency

REMEMBER **To convert inclusive classes into exclusive classes, subtract half the gap (0.5) from lower limits and add 0.5 to upper limits: 10–19 becomes 9.5–19.5. Do this before calculating median and mode.**

Sturges' rule

FORMULA **Number of classes (Sturges)**

$k = 1 + 3.322 \log_{10} N$; class width = Range ÷ k

Tabulation

Tabulation is the systematic arrangement of data in rows and columns. Parts of a table: **table number, title, head note, stub (row headings), caption (column headings), body, foot note, source note**.

4.3 Diagrams and graphs

Diagram / graph	Use
Simple bar diagram	Compare one variable across categories/years
Multiple bar diagram	Compare two or more variables side by side
Sub-divided (component) bar	Show parts of a total
Percentage bar diagram	Components as % of total
Pie diagram	Components of a total as sectors of a circle (angle = component ÷ total × 360°)
Pictogram, cartogram	Pictures / maps
Histogram	Continuous frequency distribution – adjacent rectangles; used to find **mode** graphically
Frequency polygon	Joining mid-points of histogram tops
Frequency curve	Smoothed polygon
Ogive (cumulative frequency curve)	Less-than and more-than ogives; their intersection gives the **median**; also used for quartiles
Line graph / time-series graph (historigram)	Changes over time
Lorenz curve	Inequality of income distribution

WORKED EXAMPLE Pie diagram angles

Monthly family budget: Food ₹12,000, Rent ₹6,000, Education ₹4,000, Others ₹8,000 (total ₹30,000).

Angles: Food 12/30 × 360 = 144°; Rent 72°; Education 48°; Others 96°. Total 360° ✓.

EXAM TIP Always give a title, scale, axis labels and source in diagrams – they carry marks.

REMEMBER One-liners

Pie-chart angle = (value ÷ total) × 360°. Histogram → **mode**; ogives → **median and quartiles** (the less-than and more-than ogives intersect at the median). Frequency polygon joins the mid-points of histogram tops.

Module 5: Measures of Central Tendency and Dispersion

5.1 Averages

An **average** is a single value that represents the whole data set; it lies somewhere in the centre, hence called a **measure of central tendency**.

Qualities (Yule): rigidly defined, easy to understand and calculate, based on all observations, capable of further algebraic treatment, least affected by sampling fluctuations, not unduly affected by extreme values.

FORMULA Arithmetic mean

Individual series: $\bar{X} = \Sigma X / N$

Discrete series: $\bar{X} = \Sigma fx / N$

Continuous series (direct): $\bar{X} = \Sigma fm / N$ (m = mid-value)

Shortcut: $\bar{X} = A + \Sigma fd / N$ (d = m − A)

Step deviation: $\bar{X} = A + (\Sigma fd' / N) \times h$ (d' = (m − A)/h)

Combined mean: $\bar{X}_{12} = (N_1\bar{X}_1 + N_2\bar{X}_2) / (N_1 + N_2)$

Weighted mean: $\bar{X}w = \Sigma wx / \Sigma w$

Properties of AM

- Sum of deviations from the mean is **zero**: $\Sigma(X - \bar{X}) = 0$.
- Sum of squared deviations from the mean is **minimum**.
- Combined mean can be calculated.

Merits and demerits

Merits: rigidly defined, based on all values, algebraic treatment, stable. Demerits: affected by **extreme values**, cannot be found for open-end classes without assumption, may not be an actual value (average 2.5 children).

Median is the **middle value** of the data arranged in ascending or descending order; it divides the data into two equal parts.

FORMULA Median

Individual/discrete: Median = size of (N + 1)/2 th item

Continuous: Median = $L + [(N/2 - cf) / f] \times h$

L = lower limit of median class; cf = cumulative frequency of the class before; f = frequency of median class; h = class width

Partition values: **quartiles** (Q1, Q2 = median, Q3), **deciles** (D1–D9), **percentiles** (P1–P99).

FORMULA Quartiles (continuous series)

$Q1 = L + [(N/4 - cf)/f] \times h$; $Q3 = L + [(3N/4 - cf)/f] \times h$

Merits: not affected by extreme values; can be found for open-end classes; can be found graphically (ogives). Demerits: not based on all values; not capable of algebraic treatment.

Mode is the value that occurs **most frequently** – the most typical value (e.g. most common shoe size).

FORMULA Mode (continuous series)

Mode = $L + [(f_1 - f_o) / (2f_1 - f_o - f_2)] \times h$

f_1 = frequency of modal class; f_o = frequency of preceding class; f_2 = frequency of succeeding class

Empirical relation (moderately skewed): Mode = 3 Median − 2 Mean

When frequencies are irregular, **grouping method** is used to locate the modal class. A distribution may be unimodal, bimodal or multimodal.

FORMULA GM and HM

GM = $(X_1 \times X_2 \times \ldots \times X_n)^{(1/n)}$ = antilog ($\Sigma \log X / N$)

HM = $N / \Sigma(1/X)$

For two positive numbers: AM ≥ GM ≥ HM, and $GM^2 = AM \times HM$

- **GM** – used for averaging **rates of growth**, ratios, percentages, index numbers.
- **HM** – used for averaging **speeds, rates per unit** (e.g. km per hour when distance is constant).

WORKED EXAMPLE GM and HM

(a) A company's sales grew by 10%, 20% and 32% in three years. Average growth = GM of 1.10, 1.20, 1.32 = (1.7424)^(1/3) = 1.2033 → **20.33%** (not 20.67% given by AM).

(b) A car goes 60 km at 30 km/h and returns at 60 km/h. Average speed = HM = 2/(1/30 + 1/60) = **40 km/h**.

Basis	Mean	Median	Mode
Based on all items	Yes	No	No
Affected by extremes	Yes	No	No
Open-end classes	Not possible directly	Possible	Possible
Graphical location	No	Ogive	Histogram
Algebraic treatment	Yes	No	No
Best for	Normal data, further analysis	Skewed data (income, wealth), qualitative ranks	Most common size (fashion, shoes)

COMMON MISTAKE **In the median formula, use the cumulative frequency of the class before the median class, not of the median class itself.**

FORMULA Properties used in MCQs

Sum of deviations from the mean = 0; sum of squared deviations is least about the **mean**; sum of absolute deviations is least about the **median**.

Combined mean = $(n_1\bar{x}_1 + n_2\bar{x}_2) \div (n_1 + n_2)$. If $y = a + bx$ then $\bar{y} = a + b\,\bar{x}$; median and mode change the same way.

Empirical relation: Mode = 3 Median − 2 Mean. For two numbers: $AM \times HM = GM^2$.

Quartiles: $Q_1 = (n + 1)/4$ th item; grouped data $Q_1 = L + [(N/4 - cf) \div f] \times h$.

5.2 Dispersion

Dispersion measures the **extent of scatter or variation** of individual values around the central value. Two series may have the same mean but very different spreads.

Absolute measures (in original units)	Relative measures (coefficients, unit-free)
Range	Coefficient of range
Quartile deviation	Coefficient of QD
Mean deviation	Coefficient of MD
Standard deviation	Coefficient of SD; **coefficient of variation**

Relative measures are used to **compare** variability of series with different units or averages.

√× FORMULA Range

Range = L − S (largest − smallest); Coefficient of range = (L − S)/(L + S)

Simple but based only on two extreme values. Used in **quality control** (control charts), stock prices (52-week range), weather reports.

√× FORMULA Quartile deviation

Inter-quartile range = Q3 − Q1; QD = (Q3 − Q1)/2

Coefficient of QD = (Q3 − Q1)/(Q3 + Q1)

WORKED EXAMPLE Quartile deviation

Q1 = 20, Q3 = 40. QD = (40 − 20)/2 = **10**; coefficient = 20/60 = **0.33**.

Not affected by extremes; suitable for open-end distributions; ignores 50% of data.

√× FORMULA Mean deviation

MD = $\Sigma|X - A| / N$ (A = mean or median; MD is least when taken from the median)

Discrete/continuous: MD = $\Sigma f|X - A| / N$

Coefficient of MD = MD ÷ A (the average used)

WORKED EXAMPLE Mean deviation

Data: 2, 4, 6, 8, 10. Mean = 6. |deviations| = 4, 2, 0, 2, 4 → Σ = 12. MD = 12/5 = **2.4**; coefficient = 2.4/6 = **0.4**.

Based on all values but ignores signs (not suitable for algebraic treatment).

Standard deviation (introduced by **Karl Pearson**, 1893) is the **square root of the mean of the squared deviations from the arithmetic mean**. It is the best and most widely used measure of dispersion.

√× FORMULA Standard deviation

Individual series: $\sigma = \sqrt{[\Sigma(X - \bar{X})^2 / N]} = \sqrt{[\Sigma X^2/N - (\Sigma X/N)^2]}$

Shortcut (assumed mean): $\sigma = \sqrt{[\Sigma d^2/N - (\Sigma d/N)^2]}$

Discrete/continuous: $\sigma = \sqrt{[\Sigma fd^2/N - (\Sigma fd/N)^2]}$; step deviation: multiply by h

Variance = σ^2

Combined SD: $\sigma_{12} = \sqrt{[(N_1\sigma_1^2 + N_2\sigma_2^2 + N_1d_1^2 + N_2d_2^2)/(N_1 + N_2)]}$, where $d_1 = \bar{X}_1 - \bar{X}_{12}$, $d_2 = \bar{X}_2 - \bar{X}_{12}$

Coefficient of variation (CV) = (σ / X̄) × 100

Properties of SD

- SD is **independent of change of origin** (adding a constant does not change σ) but **not of scale** (multiplying values by k multiplies σ by |k|).
- SD of the first n natural numbers = $\sqrt{[(n^2 - 1)/12]}$.
- For a normal distribution: mean ± 1σ covers **68.27%**, ± 2σ covers **95.45%**, ± 3σ covers **99.73%** of items.
- Relation (approximate, for moderately skewed data): QD ≈ (2/3)σ, MD ≈ (4/5)σ.

WORKED EXAMPLE SD and CV – individual series

Data: 2, 4, 4, 4, 5, 5, 7, 9 (N = 8). Mean = 40/8 = 5.

$(X - 5)^2$: 9, 1, 1, 1, 0, 0, 4, 16 → Σ = 32. Variance = 32/8 = 4. **σ = 2**.

CV = 2/5 × 100 = **40%**.

WORKED EXAMPLE **Which share is more stable? (CV comparison)**

Share A: mean price ₹100, σ = ₹12. Share B: mean price ₹250, σ = ₹20.

CV(A) = 12%; CV(B) = 8%. Share **B** has lower CV → **more consistent/stable**, even though its SD is larger.

WORKED EXAMPLE **SD of a frequency distribution (step deviation)**

Classes 0–10, 10–20, 20–30, 30–40, 40–50 with f = 5, 8, 15, 7, 5. Take A = 25, h = 10: d' = −2, −1, 0, 1, 2.

fd' = −10, −8, 0, 7, 10 → Σfd' = −1. fd'² = 20, 8, 0, 7, 20 → Σfd'² = 55. N = 40.

σ = 10 × √[55/40 − (−1/40)²] = 10 × √(1.375 − 0.000625) = 10 × √1.374375 = 10 × 1.1723 = **11.72 marks**.

Mean = 25 + (−1/40) × 10 = 24.75 (same as before ✓). CV = 11.72/24.75 × 100 = **47.4%**.

FORMULA **Change of origin and scale**

If y = a + bx: SD(y) = |b| × SD(x); range, QD and MD also multiply by |b|; adding a constant changes none of them. For two numbers a, b: SD = |a − b|/2. CV = σ ÷ x̄ × 100.

Module 6: Correlation and Regression

6.1 Correlation

Correlation is a statistical technique to measure the **degree and direction of relationship** between two variables.

Type	Meaning	Example
Positive	Both move in the same direction	Height and weight; advertisement and sales
Negative	Move in opposite directions	Price and quantity demanded
Linear / non-linear	Constant / changing ratio of change	–
Simple / multiple / partial	Two variables / three or more / two with others held constant	–

Methods of studying correlation

1. **Scatter diagram** – visual; shows direction and rough degree.
2. **Karl Pearson's coefficient of correlation (r)** – product-moment method.
3. **Spearman's rank correlation coefficient (R)** – for ranked/qualitative data.
4. Concurrent deviation method.

√× FORMULA Karl Pearson's coefficient of correlation

$r = \Sigma xy / \sqrt{(\Sigma x^2 \times \Sigma y^2)}$, where $x = X - \bar{X}$, $y = Y - \bar{Y}$

$r = [N\Sigma XY - \Sigma X \Sigma Y] / \sqrt{\{[N\Sigma X^2 - (\Sigma X)^2][N\Sigma Y^2 - (\Sigma Y)^2]\}}$

$r = \text{Cov}(X,Y) / (\sigma x\, \sigma y)$; $-1 \le r \le +1$

Probable error $PE = 0.6745\,(1 - r^2)/\sqrt{N}$; r is significant if $r > 6\,PE$

Coefficient of determination $= r^2$

Value of r	Interpretation
+1	Perfect positive correlation
+0.75 to +1	High positive
+0.25 to +0.75	Moderate positive
0	No (linear) correlation
−1	Perfect negative

Properties of r

Lies between −1 and +1; independent of change of origin and scale; symmetric (rxy = ryx); r is the geometric mean of the two regression coefficients.

WORKED EXAMPLE Karl Pearson's r

X: 1, 2, 3, 4, 5; Y: 2, 4, 5, 4, 5. $\bar{X} = 3$, $\bar{Y} = 4$.

x: −2, −1, 0, 1, 2; y: −2, 0, 1, 0, 1. xy: 4, 0, 0, 0, 2 → $\Sigma xy = 6$. $\Sigma x^2 = 10$; $\Sigma y^2 = 4 + 0 + 1 + 0 + 1 = 6$.

$r = 6 / \sqrt{(10 \times 6)} = 6 / 7.746 =$ **0.775** → high positive correlation.

Spearman's rank correlation

√× FORMULA Rank correlation

$R = 1 - [6\Sigma d^2 / (N(N^2 - 1))]$

With tied ranks: add $(m^3 - m)/12$ to Σd^2 for each tie

WORKED EXAMPLE Rank correlation

Two judges rank 5 contestants; $\Sigma d^2 = 6$. $R = 1 - (6 \times 6)/(5 \times 24) = 1 - 36/120 =$ **0.70**.

Correlation vs causation

High correlation does not prove cause and effect – it may be due to chance (**spurious / nonsense correlation**), a third common factor, or mutual influence.

6.2 Regression

Regression is the estimation of the unknown value of one variable from the known value of another, using the average relationship between them. The term was introduced by **Sir Francis Galton**.

FORMULA Regression equations

Y on X: $Y - \bar{Y} = byx\ (X - \bar{X})$, $byx = \Sigma xy / \Sigma x^2 = r\ (\sigma y/\sigma x)$

X on Y: $X - \bar{X} = bxy\ (Y - \bar{Y})$, $bxy = \Sigma xy / \Sigma y^2 = r\ (\sigma x/\sigma y)$

$r = \pm\sqrt{(byx \times bxy)}$ (sign same as the regression coefficients)

Both regression lines pass through $(\bar{X}, \bar{Y})$

WORKED EXAMPLE Regression equations (same data as above)

$byx = 6/10 = 0.6 \rightarrow Y - 4 = 0.6\ (X - 3) \rightarrow$ **$Y = 2.2 + 0.6X$**.

$bxy = 6/6 = 1 \rightarrow X - 3 = 1\ (Y - 4) \rightarrow$ **$X = Y - 1$**.

$r = \sqrt{(0.6 \times 1)} = 0.775$ ✓. Estimate Y when X = 6: $Y = 2.2 + 3.6 =$ **5.8**.

Correlation vs regression

Basis	Correlation	Regression
Purpose	Degree and direction of relationship	Estimating one variable from another
Variables	Both random; no dependent/independent distinction	Dependent and independent variables
Coefficient	rxy = ryx (symmetric)	byx ≠ bxy
Range	−1 to +1	Can exceed 1 (but both coefficients cannot exceed 1 together)
Units	Pure number	In units of the dependent variable
Change of scale	Independent of origin and scale	Independent of origin, not of scale

FORMULA Relations tested every term

$r = \pm\sqrt{(b_{yx} \times b_{xy})}$ (same sign as the regression coefficients); both regression lines pass through **$(\bar{x}, \bar{y})$** – solve the two lines simultaneously to get the means.

r is independent of change of origin and scale (sign changes if one scale factor is negative). Coefficient of determination = r^2.

Spearman's rank correlation: $R = 1 - 6\Sigma d^2 \div [n(n^2 - 1)]$.

WORKED EXAMPLE Means from regression lines

Lines $2x + 3y = 26$ and $x + 2y = 16$. Solving: multiply the second by 2 → $2x + 4y = 32$; subtract → $y = 6$, $x = 4$. So **$\bar{x} = 4, \bar{y} = 6$**.

Module 7: Probability

7.1 Basic terms and definitions

- **Random experiment**, **sample space** S, **event** (subset of S). **Mutually exclusive** – cannot occur together; **exhaustive** – together cover S; **equally likely**; **independent** – occurrence of one does not affect the other.
- **Classical definition:** P(A) = favourable cases ÷ total equally likely cases. $0 \le P(A) \le 1$; $P(A') = 1 - P(A)$.
- **Odds in favour** of A = m : n ⇒ P(A) = m/(m + n).

7.2 Theorems

Theorem	Formula
Addition	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$; for exclusive events P(A) + P(B)
Three events	$P(A \cup B \cup C) = \Sigma P(A) - \Sigma P(A \cap B) + P(A \cap B \cap C)$
Multiplication (independent)	$P(A \cap B) = P(A) \times P(B)$
Conditional	$P(A/B) = P(A \cap B) \div P(B)$
Compound (dependent)	$P(A \cap B) = P(A) \times P(B/A)$
Bayes' theorem	$P(A_i/B) = P(A_i)P(B/A_i) \div \Sigma P(A_j)P(B/A_j)$

7.3 Random variable and expectation

$E(X) = \Sigma x \cdot p(x)$; $E(X^2) = \Sigma x^2 \cdot p(x)$; **$Var(X) = E(X^2) - [E(X)]^2$**; $E(a + bX) = a + bE(X)$; $Var(a + bX) = b^2 Var(X)$.

WORKED EXAMPLE Probability

(a) Two dice: P(sum = 8) = 5/36; P(doublet) = 6/36 = 1/6.

(b) A card from a pack of 52: P(king or heart) = 4/52 + 13/52 − 1/52 = **4/13**.

(c) Bag with 5 red, 3 white: two drawn without replacement – P(both red) = ${}^5C_2/{}^8C_2$ = 10/28 = **5/14**.

(d) P(A) = 0.4, P(B) = 0.5, independent → P(at least one) = 1 − 0.6 × 0.5 = **0.7**.

(e) X takes 1, 2, 3 with probabilities 0.2, 0.5, 0.3 → E(X) = 0.2 + 1.0 + 0.9 = **2.1**; $E(X^2)$ = 0.2 + 2 + 2.7 = 4.9; Var = 4.9 − 4.41 = **0.49**.

(f) Leap year with 53 Sundays: 366 days = 52 weeks + 2 days → 7 equally likely pairs, 2 contain Sunday → **2/7**.

Module 8: Index Numbers and Time Series

8.1 Index numbers

An **index number** is a statistical measure of **relative change** in a variable or group of related variables over time or space, expressed as a percentage of a **base** (base = 100). Called "economic barometers".

Uses

Measuring inflation (CPI, WPI), deflating values (real income), wage negotiation and DA, business forecasting, policy formulation, stock market trends (**Sensex, Nifty**).

Methods

Method	Formula
Simple aggregative	$P_{01} = \Sigma p_1 / \Sigma p_0 \times 100$
Simple average of relatives	$P_{01} = \Sigma(p_1/p_0 \times 100) / N$
Laspeyres (base-year quantities)	$P_{01} = \Sigma p_1 q_0 / \Sigma p_0 q_0 \times 100$
Paasche (current-year quantities)	$P_{01} = \Sigma p_1 q_1 / \Sigma p_0 q_1 \times 100$
Fisher's ideal	$P_{01} = \sqrt{(\text{Laspeyres} \times \text{Paasche})}$
Dorbish–Bowley	(L + P)/2
Marshall–Edgeworth	$\Sigma p_1(q_0 + q_1) / \Sigma p_0(q_0 + q_1) \times 100$
Kelly (fixed weights)	$\Sigma p_1 q / \Sigma p_0 q \times 100$

WORKED EXAMPLE Laspeyres, Paasche and Fisher

Commodity A: $p_0 = 10$, $q_0 = 5$, $p_1 = 12$, $q_1 = 6$. Commodity B: $p_0 = 8$, $q_0 = 10$, $p_1 = 10$, $q_1 = 8$.

$\Sigma p_1 q_0 = 60 + 100 = 160$; $\Sigma p_0 q_0 = 50 + 80 = 130$; $\Sigma p_1 q_1 = 72 + 80 = 152$; $\Sigma p_0 q_1 = 60 + 64 = 124$.

Laspeyres = 160/130 × 100 = **123.08**. Paasche = 152/124 × 100 = **122.58**.

Fisher = √(123.08 × 122.58) = √15,087 = **122.83**.

Tests of adequacy

- **Time reversal test**: $P_{01} \times P_{10} = 1$.
- **Factor reversal test**: $P_{01} \times Q_{01} = \Sigma p_1 q_1 / \Sigma p_0 q_0$.
- **Fisher's index satisfies both** tests – hence called **ideal**. Laspeyres and Paasche satisfy neither.

Consumer price index (cost of living index)

FORMULA CPI methods

Aggregate expenditure method: $CPI = \Sigma p_1 q_0 / \Sigma p_0 q_0 \times 100$

Family budget method: CPI = ΣPW / ΣW, where P = price relatives, W = $p_0 q_0$ (base-year expenditure weights)

Real wage = Money wage ÷ CPI × 100; Purchasing power of money = 100 ÷ CPI

WORKED EXAMPLE Real wage

A worker's wage rose from ₹20,000 (2020) to ₹26,000 (2025); CPI rose from 100 to 125. Real wage in 2025 = 26,000 ÷ 125 × 100 = **₹20,800** → real increase only 4%, though money wage rose 30%.

Chain base index and base shifting

- **Fixed base** – all years compared with one base year. **Chain base** – each year compared with the previous year (link relatives), then chained.
- **Base shifting**: new index = old index ÷ old index of new base year × 100.
- **Splicing** connects two overlapping series.

√× FORMULA Tests and other points

Fisher's ideal index = √(Laspeyres × Paasche); it satisfies both the **time reversal** and **factor reversal** tests. Laspeyres uses base-year quantities (q_o); **Paasche uses current-year quantities (q_1)**.

Real wage = money wage ÷ price index × 100.

8.2 Time series

A **time series** is a set of observations arranged in **chronological order** (sales by year, monthly prices).

Components of time series

Component	Meaning	Example
Secular trend (T)	Long-term general tendency to increase or decrease	Population growth, rising GDP
Seasonal variation (S)	Regular, periodic changes within a year	Sale of umbrellas in monsoon, sweets at Diwali
Cyclical variation (C)	Oscillations over more than a year (business cycle – prosperity, recession, depression, recovery)	Boom and slump
Irregular (random) variation (I)	Unpredictable, erratic changes	Floods, strikes, pandemics

Models: Additive Y = T + S + C + I; Multiplicative Y = T × S × C × I.

Methods of measuring trend

1. **Free-hand (graphic) method** – simple, subjective.
2. **Semi-average method** – data divided into two halves; averages plotted and joined.
3. **Moving average method** – 3-yearly, 4-yearly (centred), 5-yearly averages smooth out fluctuations.
4. **Method of least squares** – fits a straight line Y = a + bX mathematically; best method.

√× FORMULA Least squares straight-line trend

Y = a + bX; normal equations: $\Sigma Y = Na + b\Sigma X$; $\Sigma XY = a\Sigma X + b\Sigma X^2$

If $\Sigma X = 0$ (time origin at the middle year): $a = \Sigma Y/N$; $b = \Sigma XY/\Sigma X^2$

WORKED EXAMPLE Moving average and least squares trend

Sales (₹ lakh): 2021 – 10, 2022 – 12, 2023 – 14, 2024 – 13, 2025 – 16.

3-yearly moving averages: 2022 – (10+12+14)/3 = 12.00; 2023 – (12+14+13)/3 = 13.00; 2024 – (14+13+16)/3 = 14.33.

Least squares: X = −2, −1, 0, 1, 2 (origin 2023). $\Sigma Y = 65$; $\Sigma XY = -20 - 12 + 0 + 13 + 32 = 13$; $\Sigma X^2 = 10$.

a = 65/5 = 13; b = 13/10 = 1.3 → **Y = 13 + 1.3X** (origin 2023, unit 1 year).

Trend values: 2021 – 10.4, 2022 – 11.7, 2023 – 13.0, 2024 – 14.3, 2025 – 15.6.

Forecast for 2026 (X = 3): Y = 13 + 3.9 = **₹16.9 lakh**.

Seasonal indices (brief)

Methods: simple average method, ratio-to-trend, ratio-to-moving-average, link relatives. Seasonal index of a quarter = average for the quarter ÷ grand average × 100; sum of quarterly indices = 400.

EXAM TIP **In least squares, always state the origin and the unit of X (e.g. "origin 2023, X unit = 1 year") – omitting this costs marks.**

WORKED EXAMPLE **Weighted moving average and back-calculation**

(a) 3-yearly weighted moving average with weights 1, 2, 1 for values 20, 26, 30: (20 + 52 + 30) ÷ 4 = **25.5**.

(b) The 3-yearly moving average centred on 2024 is 40; values for 2023 and 2025 are 38 and 45 → value for 2024 = 3 × 40 − 38 − 45 = **37**.